DJS3C - STATISTICAL QUALITY CONTROL

Unit - I

Quality control and need for statistical quality control techniques in industries - causes of variation - process control and product control. Specifications and tolerance limits-3σ limits, construction of Shewhart control charts - variable control charts - $\overline{X}$, R and σ charts.

Unit - II

Control charts for attributes: control chart for fraction defectives (p chart), number of defectives (d chart) and number of defects per unit (c chart).

Unit - III

Acceptance Sampling - Sampling inspection, producer's risk and consumer's risk-acceptable quality level (AQL), lot tolerance percent defective (LTPD), average outgoing quality level (AOQL), ATI and ASN. Rectifying inspection plans.

Unit - IV

Acceptance sampling by attributes: Single sampling plan - OC, AOQ, ATI and ASN curves - Double sampling plan and its advantages over single sampling plan, Operating procedure.

Unit - V

Acceptance sampling for variables-sampling plan based on normal distribution-known and unknown standard deviation cases. Determination of n and k for one- sided specification limits - OC curve.

BOOKS FOR STUDY:

1. Montgomery, D.C. (1991) Statistical Quality Control (2^{nd} Edition) John Wiley and Sons, New York.
2. Eugene L. Grant, and Richard S. Leavenworth (1988) Statistical Quality Control (Sixth Edition), McGrawhill Book co, New York.
3. Gupta, S. C. and V.K. Kapoor (1999) Fundamentals of Applied Statistics (Third Edition), Sultan Chand & sons, New Delhi.
4. Goon, A. M., M.K. Gupta and B. Dasgupta (1987) Fundamentals of Statistics, Vol. II. World Press, Kolkata.
5. Mahajan (1997) Statistical Quality Control, Dhanpat Rai & sons, New Delhi.
6. Juran, J.M.(1988) Quality Control Handbook, McGraw Hill, New York.

Unit I

Basics and Control Charts

1.1. Introduction

1.2. Basics in Statistical Quality Control

1.3. Control Charts

1.4.Variable Control Charts

1.1. Introduction

Every manufacturing organisation is concerned with the quality of its product. While it is important that quality requirements be satisfied and production schedules met, it is equally important that the finished product meet established specifications. Because, customer's satisfaction is derived from quality products and services. Stuff competition in the national and international level and consumer's awareness require production of quality goods and services for survival and growth of the company. Quality and productivity are more likely to bring prosperity into the country and improve quality of work life.

However, the management looks to achieve customer satisfaction by running its business at the desired economic level. Both these can be attained by properly integrating quality development, quality maintenance and quality improvement of the product. The integration of these three aspects of a product can be achieved through a sound quality control system.

The Meaning of "Quality"

Quality is a relative term and it is generally used with reference to the end of the product. For example, a gear used in sugarcane juice extracting machine may not possess good surface finish, tolerance and accuracy as compared with the gear used in the head stock of a lathe, still it may be considered of good quality if it works satisfactorily in the juice extracting machine. The quality is thus defined as the fitness for use/purpose at the most economical level.

The quality depends on the perception of a person in a given situation. The situation can be user-oriented, cost-oriented or supplier-oriented. Since, the item is manufactured for the use of the customer, the requirements of the customer dictates the quality of the product. Quality is to be planned, achieved, controlled and improved continuously.

The word "Quality" has variety of meanings:

1. **Fitness for purpose**

The component is said to possess good quality, if it works well in the equipment for which it is meant. Quality is thus defined as fitness for purpose.

2. **Conformance to requirements**

Quality is the ability of the material/component to perform satisfactorily in an application for which it is intended by the user. Quality of a product, thus, means conformance to requirements. Customer needs have to be assessed and translated into specifications depending upon the characteristics required for specific application. Just as every human has his own characteristics every application has its own characteristics.

An example, let us consider a fountain pen. The application of the fountain pen is to write on the paper in order to perform this function satisfactorily, the required characteristics are:

- It should hold sufficient quantity of ink, so that frequent refilling is avoided.
- It should regulate the flow of ink into the nib.
- It should mark the characters on the paper. The marking should be neither too thin nor too broad.
- It should not tear the paper.
- It should be of convenient size to hold between fingers.
- It should have a good appearance.
- It should prevent ink from drying when not in use.
- It should not be slippery nor should it hurt fingers.
- It should hold securely to the pocket.
- It should not be too expensive.
- It should have a reasonable life. It should sustain reasonable shocks (unbreakable).

Depending on these demands, it is necessary to decide the length, diameter, material, tip of the nib etc. Hence, the demands of the application are translated into the requirements and the requirements are quantified. These quantified requirements are called specifications.

3. **Grade**

Quality is a distinguishing feature or grade of the product in appearance, performance, life, reliability, taste, odour, maintainability etc. This is generally called as quality characteristic.

4. **Degree of Preference**

Quality is the degree to which a specified product is preferred over comparing products of equivalent grade, based on comparative test by customer's, normally called as customer's performance.

5. **Degree of Excellence**

Quality is a measure of degree of general excellence of the product.

6. **Measure of Fulfilment of Promises**

The quality of a product is a measure of fulfilment of the promises made to the customers.

7. In terms of product characteristics, Feigenbaum defines quality as:

“The total composite product and service characteristics of engineering, manufacturing, marketing and maintenance through which the product and service in use meet the expectation of the customers”.

The key point of this definition is that quality depends mainly on customer’s perception as described earlier. Hence, it is essential that all these features must be built in the design and maintained in manufacturing which the customer would like to have and is willing to pay for it.

For example, the product must perform its intended function repeatedly a called upon, over its stipulated life cycle under normal conditions of use. It is required that the product must look attractive and be safe in handling. It should last for a longer period and be economical. It should be easy to operate or use.

Thus we conclude that the product should have certain abilities to perform satisfactorily in a stated application. These abilities may be categorised ten factors as under:

1. **Suitability:** For specific application.
2. **Reliability:** It should give efficient and consistent performance.
3. **Durability:** It should have desired life.
4. **Workability:** Safe and fool proof workability.
5. **Affordability:** It should be economical.
6. **Maintainability:** It should be easy to maintain.
7. **Aesthetic look:** It should look attractive.
8. **Satisfaction to Customers:** It should satisfy the customer’s requirements.
9. **Economical:** It should have reasonable price.
10. **Versatility:** It should serve number of purposes.

A product can be said to possess good quality if all the above requirements are properly balanced while designing and manufacturing it.

Quality Control

Control

Control can be defined as a process by means of which we observe performance and compare it with some standard. If there is a deviation between the observed performance and the standard performance then it is necessary to take corrective action.

Quality Control

The term quality control has variety of meanings:

1. Quality control is the process through which we measure the actual quality performance, compare it with the standards and take corrective action if there is a deviation.
2. It is a systematic control of variation factors that affect the quality of the product. It depends on: Material, tools, machines, type of labour, working conditions, measuring instruments, etc.
3. Quality control can be defined as the entire collection of activities which ensures that the operation will produce the optimum quality products at minimum cost.

4. It can also be defined as the tools, devices or skills through which quality activities are carried out.
5. It is the name of the department which devotes itself full time to quality activities are carried out.
6. The procedure for meeting the quality goals is termed as quality control.
7. It is a system, plan or method of approach to the solution of quality problems.
8. Asper A. Y. Feigenbaum Total Quality Control is:

"An effective system for integrating the quality maintenance and quality improvements efforts of the various groups in an organization, so as to enable production and services at the most economical levels which allow full customer satisfaction".

Steps in Quality Control Programme

1. Formulate quality policy.
2. Work out details of product requirements, set the standards (specifications) on the basis of customer's performance, cost and profit.
3. Select inspection plan and set up procedure for checking.
4. Detect deviations from set standards or specifications.
5. Take corrective action through proper authority and make necessary changes to achieve standards.
6. Decide on salvage method i.e. to decide how the defective parts are disposed of, entire scrap or reworked.
7. Co-ordination of quality problems.
8. Developing quality consciousness in the organization. Quality control is not a function of any single department or a person. It is the primary responsibility of any supervisor to turn out work of acceptable quality.

Aims of Objectives of quality Control

1. To improve the company's income by making the product more acceptable to the customers; by providing long life, greater usefulness (versatility), aesthetic aspects, maintainability, etc.
2. To reduce company's cost through reduction of the losses due to defects. For example, to achieve lower scrap, less rework, less sorting, fewer customer returns etc.
3. To achieve interchangeability of manufacture in large scale production.
4. To produce optimum quality at minimum price.
5. To ensure satisfaction of customer's goodwill, confidence and reputation of manufactures.
6. To make inspection prompt to ensure quality control at proper stages to ensure production of non-defective products.
7. Judging the conformity of the process to the established standards and taking suitable action when there are deviations.
8. To improve quality and productivity by process control, experimentation and customers feedback.
9. Developing procedure for good vendor-vendee relations.
10. Developing quality conscious in the organisation.

Quality Characteristics

A physical or chemical property, a dimension, a temperature, pressure, taste, smell or any other requirements used to define the nature of the product or service (which contributes to fitness for use) is a quality characteristic. Thus, a metal cylinder may be defined by stating the quality characteristic contributes to fitness for use for the product.

Quality characteristic can be classified as:

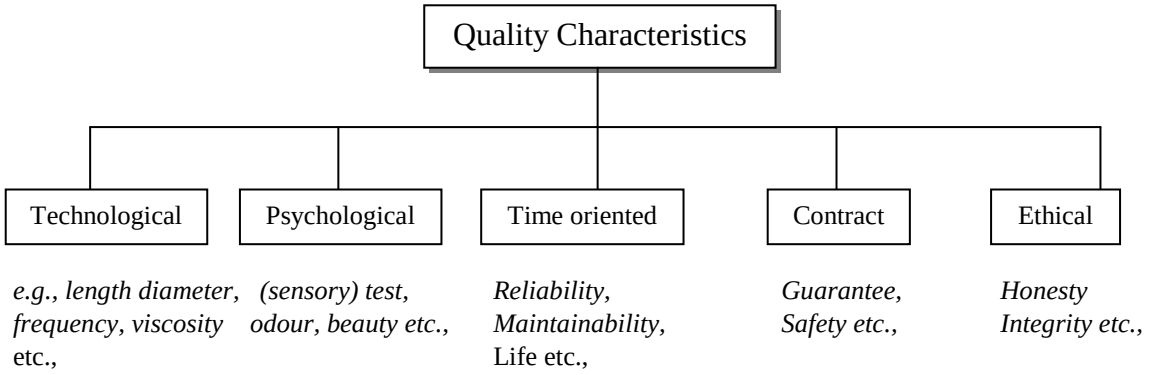

Figure: Quality Characteristics

Quality characteristics may be:

1. Directly measureable example, weight, shear strength, specific gravity, length, diameter etc.
2. Non-measurable example, rejections due to flows, cracks, breakages etc.

For each quality characteristic, there is a sequence of activities as shown in Figure:

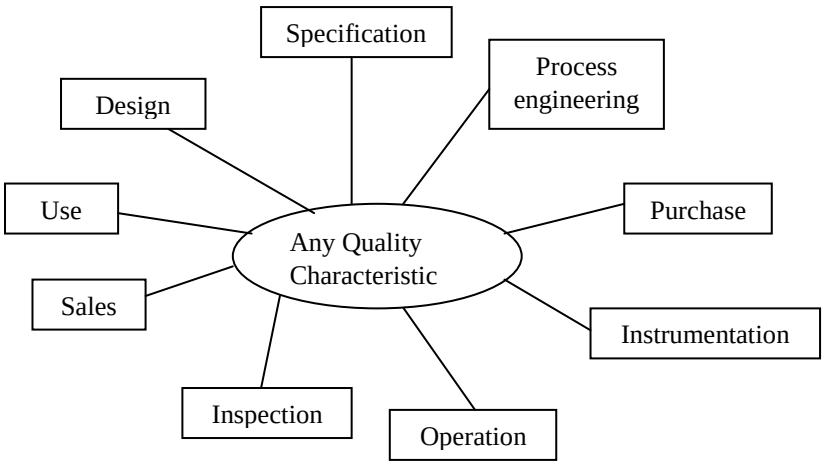

Figure: Sequence of activities for any quality characteristic.

1.2. Basics in Statistical Quality Control (SQC)

A quality control system performs inspection, testing and analysis to ensure that the quality of the products produced is as per the laid down quality standards. It is called "Statistical Quality Control". The statistical techniques are employed to control, improve and maintain quality or to solve quality problems. Statistics is the collection, organisation, analysis, interpretation and presentation of the data. It is based on law of large numbers and mathematical theory of probability. It is just one of the many tools necessary to solve quality problems it takes into account the existence of variation. Building an information system to satisfy the concept of 'prevention' and 'control' and improving upon product quality, requires statistical thinking.

SQC is systematic as compared to guess-work of haphazard process inspection and the mathematical statistical approach neutralizes personal bias and uncovers poor judgement. SQC consists of three general activities:

1. Systematic collection and graphic recording of accurate.
2. Analysing the data.
3. Practical engineering or management action, if the information obtained indicates significant deviations from the specified limits.

Modern techniques of SQC and acceptance sampling have an important part to play in the improvement of quality, enhancement of productivity, creation of consumer confidence and development of industrial economy of the country.

Relying itself on probability theory, statistical quality control plays an important role in total quality control. The following statistical tools are generally used for the purpose of exercising control, improvement of quality, enhancement of productivity, creation of consumer confidence and development of the country.

1. Frequency distribution

Frequency distribution is a tabulation or tally of the number of times a given quality characteristic occurs within the samples. Graphic representation of frequency distribution will show:

(a) Average quality
(b) Spread of quality
(c) Comparison with specific requirements
(d) Process capability

2. Control chart

Control chart is a graphical representation of quality characteristics, which indicates whether the process is under control or not.

3. Acceptance sampling

Acceptance sampling is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot on the basis of conforming or not conforming to a quality specification. It reduces the time and cost of inspection and exerts more effective pressure on quality improvement than it is possibly by 100 percent inspection.

It is used when assurance is desired for the quality of material/products either produced or received.

4. **Analysis of the data**

It includes special methods, which include such techniques as the analysis of tolerance, correlation, analysis of variance, analysis for engineering design, problem solving technique to cause of troubles.

Statistical methods can be used in arriving at proper specification limits of products, in designing the products, in the purchase of raw material, semi-finished and finished products, manufacturing processes, inspection, packaging, sales and also after sales services.

Benefits of Statistical Quality Control

1. **Efficiency:** The use of SQC ensures rapid and efficient inspection at a minimum cost.
2. **Reduction of Scrap:** It uncovers the cause of excessive variability in manufactured produced – forecasting trouble before rejections occur and reducing the amount of spoiled work.
3. Moreover, the use of acceptance sampling in SQC, exerts more effective pressure for quality improvement than is possible by 100% inspection.
4. **Easy detection of faults:** In SQC after plotting the control charts $\overline{X}, R, p, c, u, np$. When the points fall above the upper control limits or below the lower control limit it is an indication of deterioration in quality, necessary corrective action is then taken. On the other hand, with 100% inspection, unwanted variations in quality may be detected at a stage when large amount of defective products have already been produced.
5. **Adherence to specification:** So long as a statistical control continues specifications can be accurately predicted for future, by which it is possible to assess whether the production processes are capable of producing the products with the given set of specifications.
6. Increases, output and reduces wasted machine and man hours.
7. Efficient utilization of personnel, machines and materials resulting in higher productivity.
8. Better customer relations through general improvement in product and higher share of the market.
9. SQC has provided a common language that may be used, by all three groups (designers, production personnel and inspectors) in arriving at a rational solution of mutual problems.
10. Elimination of bottlenecks in the process of manufacturing.
11. Point out when and where 100 percent inspection, sorting or screening is required.
12. Creating quality awareness in employees.

However, it should be emphasized that SQC is not a panacea for assuring product quality. It simply furnished “perspective facts” upon which intelligent management and engineering action can be based. Without such action, the method is ineffective. Even the application of standard procedures without adequate study of the process is extremely dangerous.

Meaning and Scope of Statistical Quality Control

Quality has become one of the most important consumer decision factors in the selection among competing products and services. The traditional definition of quality is based on the viewpoint that products and services must meet the requirements of those who use them, that is customer's risk).

Quality means fitness for use.

Quality is inversely proportional to variability.

There are two general aspects of fitness for use

1. Quality of Design
2. Quality of conformance

All products and services are produced in various in grades or levels of quality are international and consequently, the appropriate technical term in Quality of Design.

For example, all automobiles have as their basis objective providing safe transportation for the consumer. However, automobiles differ with respect to size, appointments, appearance and performance. These differences between the types of automobiles.

The quality of conformance is how well the product conforms to the specifications required by the design. Quality of conformance is influenced by a number of factors, including the choice of manufacturing process, the training and supervision of the workforce, the type of quality assurance system used (process control, tests, inspection activates etc.), the extent to which these quality assurance producers are followed and the modification of the workforce to achieve quality.

Dimensions of Quality

Garvin (1987) provides an eight components or dimensions of quality.

1. Performance
2. Reliability
3. Durability
4. Serviceability
5. Aesthetics
6. Features
7. Perceived Quality
8. Conformance to Standards.

1. Performance (Will the product do the intended job?)

Potential customers evaluate a product to determine if it will perform certain specific functions and determine how well it performs then.

2. **Reliability (How often does the product fail?)**

Complex products, such as many applications, automobiles or airplanes will require some repair over their service life. For industry in which the customer's view of quality is greater impacted by the reliability dimension of quality.

3. **Durability (How long does the product last?)**

This is the effective service life of the product. Customers want products that perform satisfactory over a long period of time.

4. **Serviceability (How easy is it to repair the products?)**

There are many industries in which the customer's view of quality is directly influenced by how quickly and economically a repair or routine maintenance activity can be accomplished.

5. **Aesthetics (What does the product look like?)**

This is the visual appeal of the product, often taking into account factors such as style, colour, shape and other censoring features.

6. **Features (What does the product do?)**

Usually customers associate high quality with products that have added features.

7. **Perceived Quality (What is the reputation of the company or its product?)**

Customers ready on the past reputation of the company concerning quality of its products. This reputation is divert influenced by failures of the products. This reputation is visible to the public and by how the customer is treated when a quality related problem with the product is reported.

8. **Conformance to Standards (Is the product made exactly as these designer intended?)**

A high quality product exactly meets the requirements of customers. Manufactured parts that do not exactly meet the designer's requirements can cause significant quality provides when they are used as the components of a more complex assembly.

Quality Improvement

Quality improvements are the reduction of variability in process and products.

Quality Characteristic

Every product possesses a number of elements that jointly describe what the uses or consumer thinks of as quality. There parameters are often called quality characteristic.

Quality Characteristic may be of several types

1. Physical: length, weight, voltage, viscosity
2. Sensory: taste, appearance, colour
3. Time Orientation: reliability, durability, serviceability

Quality Engineering

Quality engineering is the set of operational, managerial and engineering activities that a company uses to ensure that the quality characteristics of a product are at the nominal of required levels.

A values of a measurements that corresponds to the desired values for that quality characteristic.

Note

Most organisations find it difficult and expressive to provide the customer with products that have quality characteristics that are always identical from unit to unit that match customer's expectations. A major season for this is variability there is a certain amount of variability in every product consequently. No two products are ever identical. So this variability can be described by some statistical methods. These methods plays a central role inequality improvements efforts.

- In the application of statistical method to quality. Engineering it is fairly typical to classify data on quality characteristic as either attributes or variables.
- Variable data are usually continuous measurements such as length voltage or viscosity.
- Attributes data are usually discrete data often taking the form of counts.

Continuous data involve counts (integer) for example, number of admissions, number of patients waiting, number of defective items.

Upper Specification Limit (USL)

Quality characteristic are often calculate relative to specifications for a manufactured product the specifications are the desired measurements for the quality characteristic on the components in the final product.

The larger allowable value for a quality characteristic is called USL.

Lower Specification Limit (LSL)

The smallest allowable value for a quality characteristic is called LSL.

Non-Conforming Product

A Quality Product is called non-conforming product.

Non-Conformity

A specific type of failure in the product is called a non-conformity.

Defective

A non-conforming product is considered which are non-conforming product is considered defective

Defects

If it has one or more defects, which are non-conformities that are series enough to significantly affect the safe or effective use of the product.

Quality Product

A quality product is defined as a product that meets the needs of the market place.

Quality and Improving Quality

Quality and improving quality has become an important business strategy for many organization, manufactures, distributors, transportation companies, financial services organizations, health care providers and governments agencies.

Non-conforming unit

A non-conforming unit is a unit of product that does not satisfy one or more of the specifications for that product.

Difference between defect and defective

An item is said to be defective if it fails to conform to the specifications in any of the characteristic.

Each characteristic that does not meet the specifications is a defect.

An item is defective if it contains at least one defect. For example, if a casting contains undesirable hard spots, blow holes etc., the casting is defective and the hard spots, blow holes etc. are the defects, which make the casting defective.

The np chart applies to the number of defectives in subgroups of constant size. Whereas c chart applies to the number of defects in a subgroup of constant size.

1.3. Control Chart

A control chart is an important aid or statistical device used for the study and control of the repetitive processes. It was developed by A. Shewhart and it is based upon the fact that variability does exist in all the repetitive process.

A control chart is a graphical representation of the collected information. The information may pertain to measured quality characteristics of samples.

Control Chart patterns

The Control Chart patterns can be classified into two categories

1. Chance pattern of variation (or) Common cause of variation
2. Assignable cause pattern of variation

1. Chance Pattern of Variation

A control chart having a chance pattern of variation will have the following three characteristics:

(i) Most of the points will lie near the central line.
(ii) Very few points will be near the control limits.
(iii) None of the points (except 3 in a thousand) fall outside the control limits.

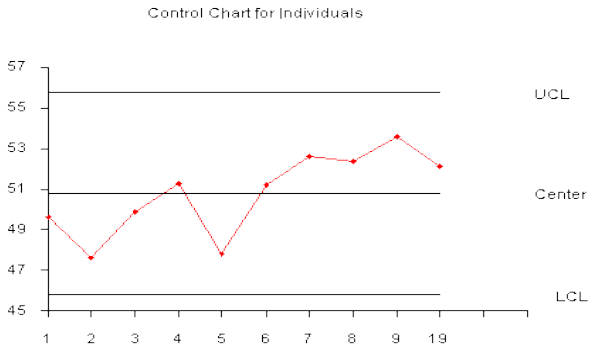

Figure: Control Chart for Chance Pattern of Variation

2. Assignable Cause Pattern of Variation

The most important types of assignable cause patterns of variations are:

(i) Extreme Variation
(ii) Indication of Trend
(iii) Shifts
(iv) Erratic Fluctuations

(i) Extreme Variation

Extreme variation is recognised by the points falling outside the upper and lower control limits. Thus, when the sample points outside these limits on $\bar{X}$ chart, p chart or both it means some assignable causes of error are present and corrective action is necessary to produce the products within the specified limits.

(a) Error in measurement and calculations
(b) Samples chosen at a peak position of temperature, pressure and such other factors.
(c) Wrong setting of machine, tools, etc.
(d) Samples chosen at the commencement or end of an operation.

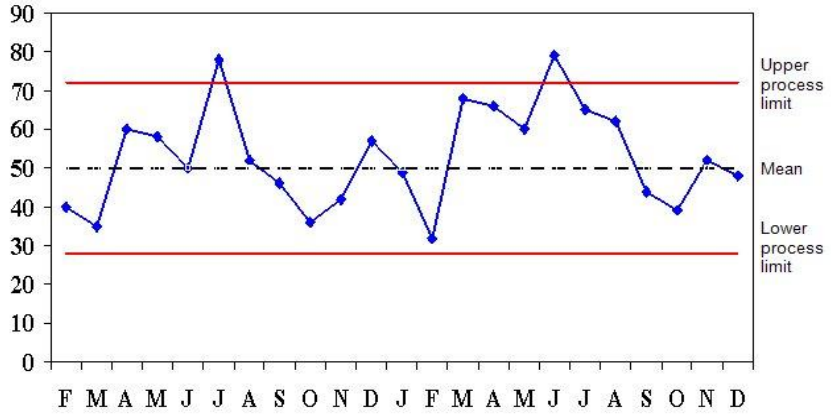

Figure: Control Chart for Extreme Variation

(ii) Indication of Trend

If the consecutive points on $\overline{X}$ or R chart tend to move steadily either towards LCL or UCL, it can be assumed that process is indicating a Trend, that is change is taking place slowly and through all the points are lying with in control limits, after some time it is likely that the process may go out of control if proper case or corrective action is not taken.

Causes of Trend

(i) Tool wear
(ii) Wear of threads on clamping device
(iii) Effects of temperature and humidity
(iv) Accumulation of dirt and clogging of fixtures and holes

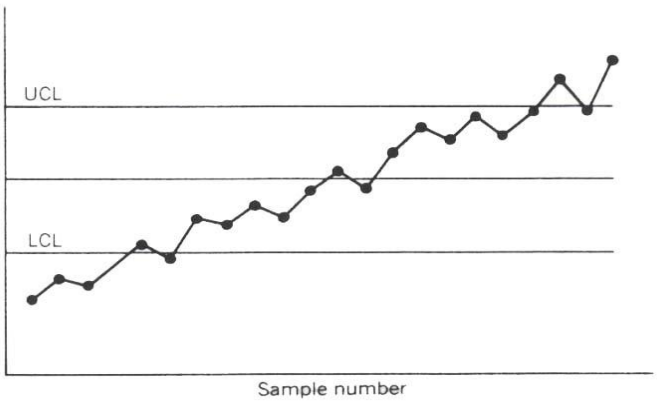

Figure: Control Chart for Indication of Trend

(iii) Shift

When a series of consecutive points fall above or below the central line on either $\overline{X}$ or R chart, it can be assumed that shift in the process has taken place indicating presence of some assignable cause.

It is generally assumed that when seven consecutive points lie above or below the central line, the shift has occurred.

Causes of Shift

(i) Change in material
(ii) Change in operator, inspector, inspection equipment
(iii) Change in machine setting
(iv) New operator, carelessness of the operator
(v) Loose fixture etc.

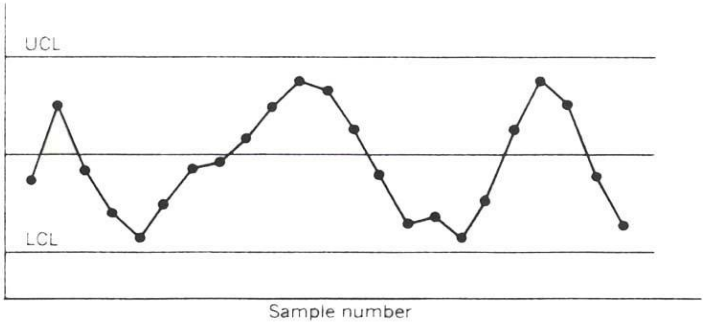

Figure: Control Chart for Shift

(iv) Erratic Fluctuations

Erratic fluctuation is characterised by ups and downs as shown in Figure is given below. This may be due to single causes or a group of causes affecting the process level and spread. The causes of erratic fluctuations are rather difficult to identify. It may be due to different causes acting at different times on the process.

Causes of Fluctuations

(i) Frequent adjustment of machine
(ii) Different types of material being processed
(iii) Change in operator, machine, test equipment etc.

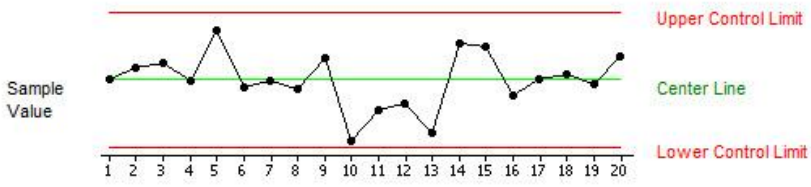

Figure: Control Chart for Erratic Fluctuations

Brief History of Statistical Quality Control

1924: W. A. Shewhart introduces the control chart concept in a Bell Laboratories Technical Memorandum.

1928: Acceptance sampling method is developed and redefined by H. F. Dodge and H. G. Roming at Bell Laboratories.

1944: Industrial quality control (a journal) beings publications.

1946: The American Society for Quality Control (ASQC) is formed as a merger of varies quality societies.

1957: Turan and Gragna: Published first time Quality Control Handbook.

1959: Technometrics (a journal of statistical for the physical, chemical engineering sureness) is established J. Stuart Hunder is the founding editor.

1969: Industrial Quality Control replaced by Quality Progress (Journal of Quality Technology).

1989: The journal Quality Engineering appears.

1989: Motorola's six sigma initiative begins.

1990: ISO 9000 certification activities increase in U. S. Industry and soon.

Basic Principles of Control Chart

A typical control chart is graphical display of a quality characteristic that has been measured or computed from a sample statistic versus the sample number or time. The chart contains center line that represents the average value of the quality characteristic corresponding to the in-control state. Two other horizontal lines called Upper Control Limit (UCL) and Lower Control Limit (LCL). These control limits are chosen so that if the process is in control, nearly all of the sample points will fall between them. As long as the points plot within the control limits, the process is assumed to be in control and no action necessary. If a point that plots outside of the control limits is interpreted as the process is out of control.

Even if all the points plot inside the control limits, if they behave in a systematic or non-random manner, then this could be an indication that the process is out of control. If the process is in control, all the plotted points should have an essentially random pattern. Usually, there is a reason why a particular non-random pattern appears on a control chart and if it can be found and eliminated process performance can be improved.

There is a close connection between control charts and hypothesis testing. If the value of $\overline{X}$ plots between the control limits, we conclude that the process mean is in control that is, it is equal to some value μ_0. On the other hand, if $\overline{X}$ exceeds either control limit, we conclude that the process mean is out of control that is, it is equal to some value $\mu_1 \neq \mu_0$. In a sense, then the control chart is a test of the hypothesis of statistical control and a point plotting outside the control limits is equivalent to rejecting the hypothesis of statistical control. But there are some differences in view point between control charts and hypothesis tests. For

example, when testing statistical hypothesis, we usually check the validity of assumptions, whereas control charts are used to detect the shifts.

One place where the hypothesis testing framework is useful is in analysing the performance of control chart. For example, we may think of the probability of type I error of the control chart (concluding the process is out of control when it is really in control) and the probability of type II error of the control chart (concluding the process is in control when it is really in control). It is occasionally helpful to use the operating characteristic curve of a control chart to display its probability of type II error. This would be an indication of the ability of the control chart to detect process shifts of different magnitudes.

In identifying and eliminating assignable causes, it is important to find the underlying root cause of the problem and to attack it.

A very important part of the corrective action process associated with control chart usage is the Out of Control Action Plan (OCAP). The OCAP contains of check points, which are potential assignable causes and terminators, which are actions taken to resolve the out of control condition hopefully by eliminating the assignable cause.

An OCAP is a living document in the sense that it will be modified over time as more knowledge and understanding of the process is gained. Consequently, when a control chart is introduced and initial OCAP should accompany it. Control charts without an OCAP are not likely to be very useful as a process improvement tool.

Control charts may be classified into two general types. Control charts for central tendency and variability are collectively called variable control charts. Many quality characteristics are not measured on a continuous scale or even a quantitative scale. In these cause, we may judge each unit of product as either conforming or non-conforming on the basis count the number of non-conformities. Control charts for such quality characteristic are called attributes control charts.

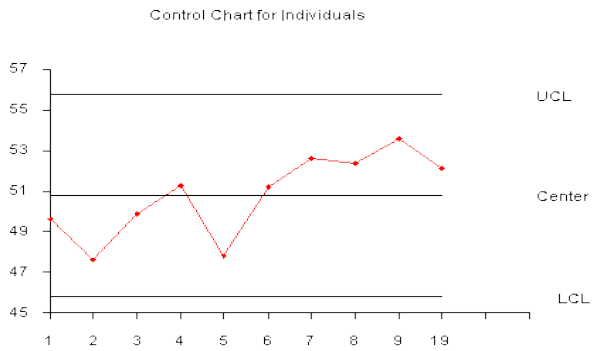

Figure: A typical control chart

Uses of Control Charts

The most important use of a control chart is to improve the process. We have found that, generally,

1. Most processes do not operate in a state of statistical control.

2. Consequently, the routine and attentive use of control charts will identify assignable causes. If these caused can be eliminated from the process, variability will be reduced and the process will be improved.
3. The control chart will only detect assignable causes. Managements, operator and engineering action will usually be necessary to eliminate the assignable causes.

Reasons for the Control Charts are popular in Industries

1. Control charts are a proven technique for improving productivity.
2. Control charts are effective in defect prevention
3. Control charts prevent unnecessary process adjustment
4. Control charts provide diagnostic information
5. Control charts provide information about process capability.

Warning Limits on Control Charts

Some analysts suggest using two sets of limits control charts. The outer limits say, at three sigma are the usual action limits, that is, when a point plots outside of this limit, a search for an assignable cause is made and corrective action is taken if necessary. The inner limits, usually at two sigma are called warning limits.

If one or more points fall between the warning limits and the control limits or very close to the warning limit, we should be suspicious that the process may not be operating properly.

The use of warning limits can increase the sensitivity of the control chart. One of their disadvantage is that they may be confusing to operating personnel.

Sample Size and Sampling Frequency

In designing a control chart, we must specify both sample size to use and the frequency of sampling. In general, larger samples will make it easier to detect small shifts in the process. If the process shift is relatively large, then we use smaller sample sizes.

We must also determine the frequency of sampling. The most desirable situation from the point of view of detecting shifts would be take large samples.

Rational Subgroups

Suppose that we are using a $\overline{X}$ control chart to detect changes in the process mean. Then the rational subgroup concept means that subgroups or samples should be selected so that if assignable causes are present, the chance for differences between subgroups will be maximized, while the chance for differences due to these assignable causes within a subgroup will be minimized.

When control charts are applied to production processes, the time order of production is a logical basis for rational sub grouping. Even though time order is preserved, it is still possible to form subgroups. Time order is frequently a good basis forming subgroups because it allows us to detect assignable causes that occur overtime.

Two general approaches are used to constructing rational subgroups. In the first approach, each sample consists of units that were produced at the same time. Ideally, we would like to take consecutive units of production. This approach is used when the primary purpose of the control chart is to detect process shifts. It minimizes the change of variability due to assignable causes within a sample and it maximizes the change of variability between samples if assignable causes are present. It also provides a better estimate of the standard deviation of the process in the process in the case of variable control charts. This approach to rational sub grouping essentially gives a snapshot of the process at each point in time where a sample is collected.

In the second approach each subgroup is a random sample of all process output over the sampling interval. This method of rational subgrouping is often used when the control chart is employed. In fact, if the process shifts to an out of control state and the back in control again between samples, it is sometimes argued that the first method of rational subgrouping defined above will be ineffective against these types of shifts and so the second method must be used.

When the rational subgroup is a random sample of all units produced over the sampling interval considerable care must be taken in interpreting the control charts. In fact, we can often make any process appear to be in statistical control just by stretching out the interval between observations in the sample.

There are other bases for forming rational subgroups. For example, suppose a process consists of several machines that pool their output into a common stream. If we sample from this common stream of output, it will be very difficult to detect whether or not some of the machines are out of control.

The rational subgroup concept is very important. The proper selection of sample requires careful information of the process, with the objective of obtaining as much useful information as possible from the control chart analysis.

Types of Control Charts

Basically control charts are classified into two types.

1. Variable Control Charts
2. Attribute Control Charts

1.4.Variable Control Chart

Variable control chart mainly consist of three charts namely

1. Mean (Average) control chart ($\overline{X}$)
2. Range control chart (R)
3. Standard deviation control chart (σ)

Attribute Control Chart

Attribute control chart mainly consist of three charts which are

1. Fraction defective chart (p)
2. Chart for defects (c)
3. Chart of number of defectives (np or d)

Assignable Causes of Variation

The assignable causes occurs different situations. They are

1. Points fall outside the control limits
2. A sequence of 7 or more points
3. Points form a trend
4. Points fall very close to central line.

Due to these causes or the presence of assignable causes, the process is considered as out of control.

Constant Cause System or Common Causes

Suppose all the points fall within the control limits but do not form a trend or any sequence of points then it is called constant cause system and the process is in control.

Process Control and Product Control

The main objective in any production process is to control and maintain the quality of the manufactured product. So that it confirms to specified quality standards. This is called process control and it is studied through control charts.

Instead of measuring the products, inspect the product one by one in such a way that, the products will accept or reject as the case may be. This is called product control and it is studied through acceptance sampling plan which has been established by Dodge and Roming.

Specification Limits and Tolerance Limits

Specification Limits

When an article is proposed to be manufactured, the manufactures have to decide upon the maximum and minimum allowable dimensions of some quality characteristics so that the product can be gainfully utilised for which it is intended. If the dimensions are beyond these limits, the product is treated as defective and cannot be used. These maximum and minimum limits of variation of individual items, as mentioned in the product design are known as ‘specification limits’.

Tolerance Limits

These are limits of variation of a quality measure of the product between which at least a specified proportion of the product is expected to lie (with a given probability), provided the process is in a state of statistical quality control. For example, we may claim with a probability of 0.99 that at least 90% of the products will have dimensions between some stated limits. These limits are also known as ‘statistical tolerance limits’.

The terms specification limits and tolerance limits are often used interchangeably. Indeed, even the American Society for Quality Control’s (ASQC) Glossary (1983) defines the two terms with one entry as “the conformance boundaries for an individual unit of a

manufacture or service operation". The Glossary does suggest that tolerance limits are generally preferred in evaluating the manufacturing or service environment, whereas specification limits are more appropriate for categorizing materials, products or services in terms of their stated requirements.

For example, a government supply agency provides specifications for mops. One of these specifications concerns the type of wood to be used for the handle. Another specification is the length of the handle as 120±2 cm. The last specification can also be considered a tolerance. Tolerance refer to physical measurements only, whereas specifications refer to characteristics, included in specifications.

Natural process limits are usually determined from population values of from large-sample estimates. Alternatively, a large number of small samples may be collected. In either case, the limits are at $\pm 3\sigma$.

Statistical tolerance limits are not to be confused with tolerance limits. Tolerance limits are set by the designers and appear on engineering drawings. Statistical tolerance limits are based on samples from the production process.

There may be lower tolerance limits (lower specification limits) and upper tolerance limits (upper specification limits). The lower tolerance limit defines the lower conformance boundary for an individual unit of a manufacturing or service operation and the upper tolerance limit applies to the upper conformance boundary.

Finally, since the ASQC Glossary equates tolerance limits to specification limits, lower tolerance limits to lower specification limits, we will also use these terms interchangeably on occasion.

Specifying Tolerances

It is practically impossible to manufacture one article exactly like another or one batch like another. Variability is one of the fundamental concepts of modern quality control. Therefore, the ranges of permissible difference in dimensions have been standardized under the name limits. The limits of size for a dimension or a part are two extreme permissible sizes for that dimension (high limit and low limit).

The difference between the high limit and the low limit which is the margin allowed for variation in workmanship is called tolerance. Tolerance can also be defined as the amount by which the job is allowed to go away from accuracy and perfectness without causing any functional trouble, when assembled with the mating part and put into actual service. Tolerance are set only on dimensions, but also on other quality characteristics as well, such as temperature, pressure and volume.

The selection of tolerance is very important. A common complaint among production personnel is that designers do not understand production problems. Inspection personnel often complain not only about the poor quality of manufactured product but also about the unreasonableness of specified tolerances. Therefore, the designers may specify one tolerance inspection gauges may allow another usually wider tolerance and foreman may be even more liberal.

3σ Limits

A. Shewhart has proposed 3σ limits to construct control charts. The distance between central line and any way one of the control limits is 3σ. Let us take a sample of size n is $(x_1, x_2, ..., x_n)$ and its statistic is taken as t. (i.e.) $E(t) = \mu_i$ *and* $V(t) = \sigma_i^2$. Thus,

$$t \sim N(\mu_i, \sigma_i^2)$$

According to normal law,

$$\Rightarrow P[-3\sigma_i \le t - \mu_i \le 3\sigma_i] = 0.9973$$
$$\Rightarrow P[|t - \mu_i| \le 3\sigma_i] = 0.9973$$

or

$$\Rightarrow P[|t - \mu_i| > 3\sigma_i] = 0.0027$$

The above probabilities states that, the probability of a random variable which goes outside 3σ limits $(\mu_i \pm 3\sigma_i)$ is considerably very small. Suppose t follows normal distribution, the observed values lie between $\mu_i + 3\sigma_i$ *and* $\mu_i - 3\sigma_i$ which are called UCL and LCL respectively. If any observed value falls outside the control limits, it is a danger signal for presenting assignable causes.

Construction of Average and Range $(\overline{X}$ *and* $R)$ Charts

The construction of Average and Range charts are based on measurements of produced goods. The measurements may be length, breath, area or volume. The selection of samples or subgroups is very essential. We select N samples in which each sample has n subgroups.

Let X_{ij} be the j^{th} observation of the i^{th} sample (i=1,2,…,N; j=1,2,...n). From the measurable data we have to calculate sample statistics such as mean $(\overline{X}_i)$, Range (R_i) and Standard deviation (S_i) of i^{th} sample,

$$\overline{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \quad (1)$$

$$R_i = \underset{j}{\text{Max}}\, X_{ij} - \underset{j}{\text{Min}}\, X_{ij} \quad (2)$$

$$s_i = \sqrt{\frac{\sum_{i=1}^{n}(X_{ij} - X_i)^2}{n}} \quad (3)$$

By using above statistics, we have to compute their averages

$$\overline{\overline{X}} = \frac{\sum \overline{X}_i}{N} \quad (4)$$

$$\overline{R} = \frac{1}{N}\sum_{i=1}^{N} R_i \tag{5}$$

$$\bar{s} = \frac{1}{N}\sum_{i=1}^{N} s_i \tag{6}$$

After finding the averages of the statistics, we have to frame the control limits for framing a limits we have to calculate the value of 3σ, where σ is the standard deviations of the universe. The standard error of i^{th} subgroup is defined

$$SE(\overline{X}_i) = \frac{\sigma}{\sqrt{n}}, i = 1,2,...,N \tag{7}$$

From the sampling distribution of range,

$$E(R) = \overline{R} = d_2\sigma \tag{8}$$

$$\Rightarrow \sigma = \frac{\overline{R}}{d_2} \tag{9}$$

where d_2 is a constant relative to the subgroup size.

Control chart for average is constructed when,

(i) μ and σ are unknown
(ii) μ and σ are known
(iii) R is unknown

Case (i): μ and σ are unknown

Suppose the population mean (μ) and population standard deviation (σ) are not given we have to calculate the sample mean and sample standard deviation or sample range.

Let $\overline{X}_1, \overline{X}_2, ..., \overline{X}_N$ be averages of subgroups. The average of averages is estimated by using the formula given in equation (4). The required 3σ control limits is defined as

$$E(\overline{X}_i) \pm 3SE(\overline{X}_i) = \overline{\overline{X}} \pm \frac{3\sigma}{\sqrt{n}} \quad \text{(using (7))}$$

$$= \overline{\overline{X}} + \frac{3}{\sqrt{n}}\frac{\overline{R}}{d_2} \quad \text{(using equation (9))}$$

$$= \overline{\overline{X}} + \left(\frac{3}{\sqrt{n}}d_2\right)\overline{R}$$

$$= \overline{\overline{X}} \pm A_2\overline{R} \tag{10}$$

Here A_2 is also a constant which is obtained from a table containing the factors for control charts and it depends on its subgroup size n.

The equation (10) is re-written as

$$UCL = \overline{\overline{X}} + A_2\overline{R} \tag{11}$$

$$LCL = \overline{\overline{X}} - A_2\overline{R} \tag{12}$$

By using equation (4), we draw a horizontal line parallel to x - axis and it represents the central line of the chart. $\therefore CL = \overline{\overline{X}}$. Similarly using equations (11) and (12), we draw dotted horizontal lines and they represent upper control limit and lower control limit respectively.

Using subgroup averages (equation (1)) we plot the points and infer that weather the process is in control or out of control. If the process is out of control do the necessary steps and draw the process is in control.

Case (ii): μ and σ are known

In the case of known population constants we have to calculate sample subgroup means and standard deviation. We have

$$E(\overline{X}_i) \pm 3SE(\overline{X}_i) = \mu \pm 3\frac{\sigma}{\sqrt{n}}$$

$$= \mu \pm \frac{3}{\sqrt{n}}\sigma$$

$$= \mu \pm A\sigma \tag{13}$$

where A is a constant depends on n. The required control limits are,

$$UCL = \mu + A\sigma \tag{14}$$

$$LCL = \mu - A\sigma \tag{15}$$

Case (iii): R is unknown

In the case of unknown range we have to construct $\overline{X}$ chart by using another statistic called standard deviation. The standard deviation of i^{th} subgroup s_i (given in (3)) is calculated and also the average of standard deviation is computed by using

$$\overline{S} = \frac{1}{N}\sum_{i=1}^{N} s_i$$

We know that the relation between average of the sample standard deviations and population standard deviations.

$$\overline{S} = C_2\sigma$$

$$\Rightarrow \sigma = \frac{\overline{S}}{C_2} \tag{16}$$

Control limits are defined as

$$E(\bar{X}_i) \pm 3SE(\bar{X}_i)$$

$$= \bar{\bar{X}} \pm 3\frac{\sigma}{\sqrt{n}}$$

$$= \bar{\bar{X}} \pm \frac{3}{\sqrt{n}}\frac{\bar{S}}{C_2}$$

$$= \bar{\bar{X}} \pm \left(\frac{3}{\sqrt{n}C_2}\right)\bar{S}$$

$$= \bar{\bar{X}} \pm A_1\bar{S} \quad (17)$$

The required control limits are

$$UCL = \bar{\bar{X}} + A_1\bar{s} \quad (18)$$

$$LCL = \bar{\bar{X}} - A_1\bar{s} \quad (19)$$

$$CL = \bar{\bar{X}} \quad (20)$$

here A_1 is also a constant depends on n. By using the value of $\bar{\bar{X}}$, we draw a horizontal line parallel to x-axis and it is named as central line of the chart (20). The upper and lower control limits are drawn are dotted horizontal lines by using the equations (18) and (19) respectively.

The values of subgroup averages are plotted in the chart and verified the process is in control or not.

Control Limits for R Chart

Let X_{ij} be j^{th} observation in i^{th} subgroup (i=1, 2, …, N; j=1, 2, …, n). We have to find range for each subgroup. The range of i^{th} subgroup is,

$$R_i = Max(X_{ij}) - Min(X_{ij}) \quad (i = 1,2,...,N)$$

The average of ranges is

$$\bar{R} = \frac{1}{N}\sum_{i=1}^{N} R_i$$

Control limits for R chart are defined as

$$E(R) \pm 3SE(R)$$

$$= \bar{R} \pm 3C\bar{R}$$

$$= (1 \pm 3C)\bar{R}$$

Here

$$E(R) = \bar{R}$$

$$SE(R) = \sigma R$$

We know that, $\sigma R = C.\ E(R)$

$$= C\overline{R}$$

$$SE(R) = \sigma R = C\overline{R}$$

Hence, the limits are

$$UCL = (1 + 3C)\overline{R} = D_4\overline{R}$$

and

$$LCL = (1 - 3C)\overline{R} = D_3\overline{R}$$

Here D_3 and D_4 are constants taken from the table containing the factors of control charts defending on the subgroup size.

If the subgroup size is less than 7, D_3 becomes zero. In this case we obtain only upper control limit. As in the case of $\overline{X}$ chart, we draw central line by using the value of $\overline{R}$ as a bold horizontal line and the upper control limit is a drawn as dotted horizontal line using in the value of $D_4\overline{R}$. After drawing in central line and control limit we plot the values of ranges. Finally we concluded that the process is in control or not. Suppose a subgroup size $n \geq 7$, lower control limit is also drawn as dotted horizontal line.

Control Chart for Standard Deviation σ Chart

Let X_{ij} be j^{th} observation of the i^{th} sample. Let S and σ be the standard deviations of sample and population. Control chart for standard deviation is constructed under the condition that when σ is unknown and σ is known.

Case (i): σ is unknown

Suppose the population standard deviation is not known, we have to calculate sample standard deviation for constructing standard deviation chart. Let S_i be the standard deviation of i^{th} subgroup

$$S_i = \sqrt{\frac{\sum (X_{ij} - \overline{X}_i)^2}{n}}$$

The average of the standard deviation is obtained as

$$\overline{S} = \frac{1}{N}\sum_{i=1}^{N} S_i$$

We know that, the relation between population standard deviation and average of the sample standard deviation is

$$\overline{S} = C_2\sigma$$

The control limits of standard deviation chart are

$$= E(S) \pm 3SE(S)$$

$$= \bar{S} \pm 3(C_3\sigma)$$

$$= \bar{S} \pm 3C_3\left(\frac{\bar{S}}{C_2}\right)$$

$$= \bar{S} \pm \left(3\frac{C_3}{C_2}\right)\bar{S}$$

Hence,

$$UCL = \left(1+3\frac{C_3}{C_2}\right)\bar{S} = B_4\bar{S}$$

$$LCL = \left(1-3\frac{C_3}{C_2}\right)\bar{S} = B_3\bar{S}$$

After drawing central line and control limits, plot the points by using subgroup standard deviation and draw suitable conclusion. If the value of lower control limit is negative, we take the value as zero. Here the values of D_3 and D_4 are taken from the pre-assigned table depending on subgroup size n.

Case (ii): σ is known

Suppose the population standard deviation is known, there is no need to compute sample statistic, now we define the following:

$E(S) = C_2\sigma$ and

$SE(S) = C_3\sigma$.

The control limits of standard deviation chart are

$$= E(S) \pm 3SE(S)$$

$$= C_2\sigma \pm 3C_3\sigma$$

$$= (C_2 \pm 3C_3)\sigma$$

Hence,

$$UCL = (C_2 + 3C_3)\sigma = B_2\sigma$$

$$LCL = (C_2 - 3C_3)\sigma = B_1\sigma$$

$$Central\ Line = C_2\sigma.$$

Problem 1:

Control charts for $\overline{X}$ and R are maintained on certain dimensions of a manufactured part, measured in mm. The subgroup size is 4. The values of $\overline{X}$ and R are computed for each subgroup. After 20 subgroups $\sum \overline{X} = 412.83 and \sum R = 3.39$. Compute the values 3 sigma limits for the $\overline{X}$ and R charts and estimate the value of σ' on the assumption that the process is in statistical control.

Solution:

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{N}$$

Where N=number of subgroups

Therefore,

$$\overline{\overline{X}} = \frac{412.83}{20} = 20.6415$$

$$\overline{R} = \frac{\sum R}{N}$$

$$\overline{R} = \frac{3.39}{20} = 0.169$$

i.e. σ' =population standard deviation

$$\sigma' = \frac{\overline{R}}{d_2} = \frac{0.169}{2.059} = 0.082$$ [for subgroup of factor d_2=2.059]

$$3\sigma_{\overline{X}} = \frac{3\sigma'}{\sqrt{n}} = \frac{3 \times 0.082}{\sqrt{4}} = 0.123$$

For $\overline{\overline{X}}$ chart:

$$UCL_{\overline{X}} = \overline{\overline{X}} + 3\sigma_{\overline{X}}$$

= 20.6415+0.123

= 20.7645

$$LCL_{\overline{X}} = \overline{\overline{X}} - 3\sigma_{\overline{X}}$$

= 206415-0.123

= 20.5185

For R chart:

$$UCL_R = D_4 \overline{R}$$

= 2.28*0.169 [for subgroup of 4 factor D_4=2.28 from table]

= 0.3853

$LCL_R = D_3\overline{R}$

= 0*0.169 [for subgroup of 4, D_3=0]

= 0.

Problem 2:

In a capability study of a lathe used in turning a shaft to a diameter of 23.75±0.1 mm a sample of 6 consecutive pieces was taken each day for 8 days. The diameters of these shafts are as given below:

1st Day	2nd Day	3rd Day	4th Day	5th Day	6th Day	7th Day	8th Day
23.77	23.80	23.77	23.79	23.75	23.78	23.76	23.76
23.80	23.78	23.78	23.76	23.78	23.76	23.78	23.79
23.78	23.76	23.77	23.79	23.78	23.73	23.75	23.77
23.73	23.70	23.77	23.74	23.77	23.76	23.76	23.72
23.76	23.81	23.80	23.82	23.76	23.74	23.81	23.78
23.75	23.77	23.74	23.76	23.79	23.78	23.80	23.78

Construct the $\overline{X}$ and R chart and find out the process capability for the machine.

Solution:

Average diameter for the first day

$$\overline{X}_1 = \frac{X_1 + X_2 + X_3 + X_4 + X_5 + X_6}{6}$$

$$= \frac{23.77 + 23.80 + 23.78 + 23.73 + 23.76 + 23.75}{6} = 23.765$$

Similarly, the average for each day are calculated and the results are tabulated as below:

$\overline{X}_1$	$\overline{X}_2$	$\overline{X}_3$	$\overline{X}_4$	$\overline{X}_5$	$\overline{X}_6$	$\overline{X}_7$	$\overline{X}_8$
23.765	23.77	23.7716	23.7767	23.7717	23.7583	23.7767	23.7667

Now,

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{N}$$

$$= \frac{190.1567}{8} = 23.7696.$$

Ranges:

R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
0.07	0.11	0.06	0.08	0.04	0.05	0.06	0.07

$$\bar{R} = \frac{\sum R}{N} = 0.0675$$

For $\bar{X}$ chart:

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$

= 23.7696+0.48*0.0675 [A_2 = 0.48 for subgroup of from table appendix]

= 23.802

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

= 23.7696-0.0324

= 23.7322

For R chart:

$$UCL_R = D_4\bar{R}$$

= 2*0.0675

= 0.1350

$LCL_R = D_3\bar{R} = 0$ [D_3=0 for subgroup of 6 or less]

Process capability:

$$6\sigma' = 6 \times \frac{\bar{R}}{d_2} = \frac{6 \times 0.0675}{2.534} = 0.15982 \quad [for\ subgroup\ of\ 6, d_2 = 2.534\ from\ table\ appendix]$$

$X_{\max} - X_{\min} = 0.2\,mm$ from data.

Therefore, $(X_{\max} - X_{\min}) > 6\sigma'$.

So, we conclude that all manufactured products will meet specifications as long as the process stays in control.

Problem 3:

The following table shows the average and ranges of the spindle diameters in millimetres for 30 subgroups of 5 items each.

$\bar{X}$	**R**	$\bar{X}$	**R**	$\bar{X}$	**R**
45.020	0.375	45.600	0.275	45.26	0.150
44.950	0.450	45.020	0.175	45.650	0.200
45.480	0.450	45.320	0.200	45.620	0.400
45.320	0.150	45.560	0.425	45.480	0.225
45.280	0.200	45.140	0.250	45.380	0.125
45.820	0.250	45.620	0.375	45.660	0.350
45.580	0.275	45.800	0.475	45.460	0.225
45.400	0.475	45.500	0.200	45.640	0.375
45.660	0.475	45.780	0.275	45.390	0.650
45.680	0.275	45.640	0.225	45.290	0.350

For the first 20 samples set up on $\bar{X}$ chart and R an R chart. Plot the next 10 samples on these charts to see if the process continues "under control" both as to average and range. Also find the process capability.

Solution:

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{N} = \frac{909.170}{20} = 45.4585$$

$$\bar{R} = \frac{\sum R}{N} = \frac{6.250}{20} = 0.3125$$

$$\sigma' = \frac{\bar{R}}{d_2} = \frac{0.3125}{2.326} = 0.13435$$

$$\sigma_{\bar{X}} = \frac{\sigma'}{\sqrt{n}} = \frac{0.13435}{\sqrt{5}} = 0.06009$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}}$$

$$= 45.4585+3*0.06009$$

$$= 45.4585+0.1803$$

$$= 45.6388$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{X}}$$

$$= 45.4585-0.1803$$

$$= 45.2782$$

$$UCL_R = D_3\bar{R}$$

$$= 2.11*0.3125$$

$$= 0.6594$$

$$LCL_R = D_3\bar{R} = 0$$

Process Capability:

$$6\sigma' = 6 \times 0.13425 = 0.80550$$

Exercises

1. Define statistical quality control.
2. What are the causes of quality variation?
3. What are the applications of control chart?
4. A sub-group of 5 items each are taken from a manufacturing process at a regular interval. A certain quality characteristic is measured and $\bar{X}$ and R values computed. After 25 subgroups it is found that $\sum \bar{X} = 357.50 \, and \sum R = 8.80$. If the specification limits are 14.40±0.40 and if the process is in statistical control, what conclusions can you draw about the ability of the process to produce items within specifications?
   (For subgroup of 5 items, $d_2 = 2.326$).
5. Explain the meaning and scope of Statistical Quality Control.
6. Explain (i) 3σ limits (ii) control limits.
7. Explain the construction and operations of $\bar{X}$ and R chart.
8. Explain the concept of rational of rational subgroups, specification, tolerance and warning limits.
9. Determine the control limits for $\bar{X}$ and R charts if $\sum \bar{X} = 357.50 \, and \sum R = 9.90$, Number of subgroups = 20. It is given that $A_2 = 0.18, D_3 = 0.41,$ $D_4 = 1.59$ and $d_2 = 3.735$. Also find the process capability.
10. (a) Differentiate between the Chance causes and Assignable causes of variation giving suitable examples.
    (b) What is meant by natural tolerance of process?
11. Explain the factors to be considered in determining

    (a) Sample size.

    (b) Frequency of subgrouping.

    (c) Basis of subgrouping.

Unit II

Control Charts for Attributes

2.1. Introduction

2.2. Control chart for fraction defectives (p – chart)

2.3. Control chart for number of defectives (np or d – chart)

2.4. Control chart for number of defects per unit (c – chart)

2.5. Control chart for number of defects in variable sample size (u – chart)

2.1. Introduction

Average and Range charts are very powerful statistical techniques to point out the troubles in the production process. Variable charts are drawing based on measurable units. They do not help to study the quality characteristics of the products. For analysing the quality characteristics of the products, Shewhart has established another set of control charts which are called attribute control charts and they are given below:

1. p – chart: control chart for fraction defective
2. np – chart: control chart for number of defectives
3. c – chart: control chart for number of defects
4. u – chart: control chart for number of defects in variable sample size.

2.2. Control chart for fraction defectives (p – chart)

In production process, inspection is carried out for identified conformity and non-conformity units. Let d be the number of non-conformity is a sample of size n. Let p be the sample proportional defective and it is defined as the ratio of the number of non-conformity units to the total number of units inspected. That is,

$$P = \frac{d}{n}$$

The corresponding population proportion is taken as P according to Binomial law the number of non-conformities,

$$d \sim B(n, P)$$
$$\Rightarrow E(d) = nP$$

and v(d) = nPQ, where Q = 1-P for constructing control limits,

$$\mathrm{v(p)} = \frac{1}{\mathrm{n}^2}\,\mathrm{v(d)}$$

$$= \frac{1}{n^2} nPQ$$

$$v(p) = \frac{PQ}{n}$$

Control limits are defined as

$$E(p) = E\left(\frac{d}{n}\right)$$

$$= \frac{1}{n} E(d)$$

$$= \frac{1}{n} np$$

$$E(p) = P$$

$$E(p) \pm 3SE(p)$$

$$= P \pm 3\sqrt{\frac{PQ}{n}}$$

$$= P \pm A\sqrt{PQ}, \quad \text{where} \quad A = \frac{3}{\sqrt{n}}$$

Case (i): Standards are known

Let P' be the value of P then the control limits are,

$$P' \pm A\sqrt{P'(1-P')} \ \ and \ \ CL = P'$$

Case (ii): Standards are unknown

(a) Sample size varies:

Consider k samples of different sizes $n_1, n_2, ..., n_k$ the sample units are inspected and the number of defective units are obtained as $d_1, d_2, ..., d_k$ respectively. The fraction defective is defined as,

$$P_i = \frac{d_i}{n_i}, \quad i = 1,2,...,k$$

The average with the fraction defectives in computed as

$$\overline{P} = \frac{\sum p_i}{k}$$

In other words, $\overline{p}$ is computed as,

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{\sum n_i p_i}{\sum n_i}$$

$\bar{p}$ indicates the central line. The control limits are

$$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n_i}$$

Plot the value of p_i in the chart and in found that the process is in control or not. If any point falls outside the control limits, remove that point and construct revised control chart for the remaining observations.

(b) Sample size is fixed

Consider k samples of equal size n inspect each and every sample and the defective units are denoted as $d_1, d_2, ..., d_k$ the fraction defective of i^{th} sample defined as the ratio of the number of defective units in the i^{th} sample (d_i) to the sample size.

$$p_i = \frac{d_i}{n},\ i = 1,2,...,k$$

The average of the fraction defectives is computed by using the relation

$$\bar{p} = \frac{\sum d_i}{k(n)}$$

$\bar{p}$ indicates the central line. Then the control limits are,

$$\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n}$$

After drawing the control limits and central line, we have to plot the values of p_i in the chart and draw the conclusion whether the process is in control or not. Suppose any point falls outside the control limits, we remove that points and construct the revised control chart.

2.3. Control chart for number of defectives (np or d – chart)

In a production process, the number of non-conformities are collected after expecting the sampling units. Let d be the number of defective units. It is noted that the proportion of defectives is given by the relation p = d/n, this implies that,

$$E(p) = P$$

The mean and variance of non-conformities units for obtained in terms of population proportion

$$\text{Mean} = E(d) = nP$$

$$\text{Variance} = v(d) = nP(1-P)$$

$$\Rightarrow SE(d) = \sqrt{nP(1-P)}$$

Hence, central line = nP.

Control limits are

$$E(d) \pm 3SE(d)$$

$$= nP \pm 3\sqrt{nP(1-P)}$$

$$\Rightarrow UCL = nP + 3\sqrt{nP(1-P)}$$

$$LCL = nP - 3\sqrt{nP(1-P)}$$

After drawing central line and control limits, we have to plot the values of number of defectives (d_i). Suppose any point falls outside the limits, we have to find out the reasons and rectified the causes.

2.4. Control chart for number of defects per unit (c – chart)

In any production process inspections is carried out for supporting standard items and bad items (defective items) by considering defectives, p chart and np chart are appropriate charts.

Suppose one may try to study about the defects per unit, another control chart, namely c chart is used. According to probability law, number of defects per unit follows Poisson distribution. Generally the population average for number of defects is denoted as C.

$$X \sim P(C)$$

$$P(x) = e^{-C}\frac{C^x}{x!}$$

Here, we known that X is the number of non-conformities and C is the parameter.

Case (i): Standards known

Let C be the average of number of defects in the population. C is a Poisson parameter and the control limits are defined as,

$$C \pm 3\sqrt{C}$$

C indicates the central line.

Case (ii): Standards unknown

From industrial products, collect defective pieces. Inspect each and every defective pieces and noted the number of defects per product. Let $c_1, c_2, \ldots, c_n$ be the number of defects and its average is

$$\overline{C} = \frac{\sum_{i=1}^{n} C_i}{n}$$

This $\bar{C}$ is also follows Poisson distribution and it represents the central line of the chart. We required control limits are

$$\bar{C} \pm 3\sqrt{\bar{C}}$$

After drawing central line and control limits, plot the values of $c_1, c_2, ..., c_n$ and conclude that the process is in control or not.

2.5. Control chart for number of defects in variable sample size (u – chart)

This control chart is different from c chart. In c - chart, each and every unit is taken as a sample. Suppose we have many number of defective units. There is no possibility to construct the c-chart, we apply u chart, which is otherwise called c chart for variable size.

Let $n_1, n_2, ..., n_k$ be sizes of different samples (or) the sampling units are considered as defective units. Observe the number of defective in each unit and count the total defects in each sample. Let $c_1, c_2, ..., c_k$ be the number of defects of the above samples $n_1, n_2, ..., n_k$ respectively. The ratio of the number of defects (C_i) to the number of defective units (n_i) is taken as,

$$u_i = \frac{c_i}{n_i}, \quad i = 1,2,...,k$$

Now compute the average of $u_i^{'}s$

$$\bar{u} = \frac{\sum_{i=1}^{k} u_i}{k}$$

It is noted that $\bar{u}$ follows Poisson distribution. According to a standard error of sample mean, we have

$$SE(\bar{u}) = \sqrt{\frac{\bar{u}}{n_i}}$$

The control limits are defined as

$$E(u_i) \pm 3SE(u_i)$$

$$\bar{u} \pm 3\sqrt{\frac{\bar{u}}{n_i}}$$

Here, $\bar{u}$ indicates the central line. After drawing control limits, plot the values of $u_i^{'}s$ corresponding to sample numbers.

Comparison of $\overline{X}$ and R chart with p chart

1. p chart is attribute control chart, i.e. for quality characteristic that can be classified as either conforming or nonconforming to the specifications. For example, dimensions checked by Go-No-Go gauges. Whereas, $\overline{X}$ and R chart is used for quality characteristic that can be measured and expressed in numbers.
2. The cost of collecting the data for p chart is less than the cost of collecting the data for $\overline{X}$ and R chart. For example, 10 shafts might be inspected with "go-no-go" gauge in the time required to measure a single shaft diameter with a micrometer. Secondly, p chart uses data already collected for other purpose.
3. The cost of computing and changing may also be less since p chart can be applied to any number of quality characteristics observed on one article. But separate $\overline{X}$ and R chart is required for each measured quality characteristic, which may be impracticable and uneconomical.
4. p chart is best suited in cases where inspection is carried out with a view to classifying an article as accepted or rejected. $\overline{X}$ and R charts are best suited for critical dimensions.
5. p chart though discloses the presence of assignable causes of variations, it is not as sensitive as $\overline{X}$ and R chart. For actual diagnosis of causes of troubles, $\overline{X}$ and R charts are best, still p chart can be used effectively in the improvement of quality.
6. The sample size is generally larger for p chart than for $\overline{X}$ and R chart. The variations in the sample size influence the control limits much more in $\overline{X}$ and R charts than in p chart.
7. The control chart for fractions defective provides management with a useful record of quality history.

Purpose of the p- chart

Because of the lower inspection and maintenance costs of p charts, they usually have a greater area of economical applications than do the control charts for variables. A control chart for fraction defective may have any one or all of the following purposes:

1. To discover the average proportion of defective articles submitted for inspection, over a period of time.
2. To bring to the attention of the management, any changes in average quality level.
3. To discover, identify and correct causes of bad quality.
4. To discover, identify and correct the erratic causes of quality improvement.
5. To suggest where it is necessary to use $\overline{X}$ and R charts to diagnose quality problems.
6. In a sampling inspection of large lots of purchased articles.

Basis of control limits on c- chart

Control limits on c chart are based on Poisson distribution. Therefore, two conditions must be satisfied.

- The first condition specifies that the area of opportunity for occurrence of defects should be fairly constant from period. The expression may be in terms of defects per unit being employed. For example, while inspecting the imperfections of a cloth it is necessary to take some units area say 100 square meters and count the number of imperfections per unit (i.e. per 100% square meters). Another example, may be number of point's imperfections per square area of painted surface. However, c chart need not be restricted to a single type of defect but may be applicable for the total of many different kinds of defects observed on any unit.
- Second condition specifies that opportunities for defects are large, while the changes of a defect occurring in anyone spot are small. For example, consider a case in which the product is large unit, say a ratio, which can have defects at number of points although any one point has only few defects.

Comparison between attribute charts and variable charts

Choosing a particular type of chart is a question of balancing the cost of collecting and analysing the type of data required to plot the chart against usefulness of the conclusions that can be drawn from the chart.

	Variable Charts	Attribute Charts
1.	**Example** $\overline{X}$, R, σ charts.	p, np, c, u charts.
2.	**Type of Data Required** Variables data (Measured values of characteristics).	Attribute data (using Go-No-Go gauges).
3.	**Filed of Application** Control of individual characteristics.	Control of proportion of defectives or number of defects or number of defects per unit.
4.	**Advantages** ➢ Provides maximum utilisation of information available from data. ➢ Provides detailed information on process average and variation for control of individual dimensions.	➢ Data required are often already available from inspection records. ➢ Easily understood by all persons. Since, it is more simple as compared to $\overline{X}$ and R chart. ➢ It provides overall picture of quality history.
5.	**Disadvantages** ➢ They are not easily understood unless training is provided. ➢ Can be confusion between control limits and specification limits.	➢ They do not provide detailed information for control of individual characteristic. ➢ They do not recognise different degree of defectiveness.

	➢ Cannot be used with go-no-go gauge inspection.	(Weightage of defects).

Generally compromise is made, it is usual to start with p chart and only for those cases shown out of control on p chart, $\overline{X}$ and R chart are plotted for detailed analysis.

Problem 1:

Following are the inspection results of magnets for nineteen observations

Week Number	Number of magnets inspection	Number of defective magnets	Fraction defective
1	724	48	0.066
2	763	83	0.109
3	748	70	0.094
4	748	85	0.114
5	724	45	0.062
6	727	56	0.077
7	726	48	0.066
8	719	67	0.093
9	759	37	0.049
10	745	52	0.070
11	736	47	0.064
12	739	50	0.068
13	723	47	0.065
14	748	57	0.076
15	770	51	0.066
16	756	71	0.094
17	719	53	0.074
18	757	34	0.045
19	760	29	0.038
Total	14091	1030	

Calculate the average fraction defective and 3 sigma control limits, construct the control chart and state whether the process is in statistical control.

Solution:

The average sample size

$$= \frac{14091}{19} = 741.63 \approx 742$$

The average fraction defectives

$$\bar{p} = \frac{\text{Total defectives in all samples}}{\text{Total inspected in all samples}} = \frac{1030}{14091} = 0.0731$$

$$UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0.0731 + 3\sqrt{0.0731(1-0.00731)/742}$$

$$= 0.1018$$

$$LCL_p = \bar{p} - 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0.0731\text{-}0.0287$$

$$= 0.0444$$

We concluded that from the resulting control chart, sample numbers 2nd and 4th are going above the upper control limits and the sample number 19th goes below the lower control limit. Therefore the process does not exhibit statistical control.

Problem 2:

A certain product is given 100% inspection as it is manufactured and the resultant data are summarized by the hour. In the following table, 156 hours of data are recorded. Calculate the control limits using 3 sigma control limits and indicate the values that are out of control.

Hour	Number of units inspected	Number of defective units	Fraction defective
1	48	5	0.104
2	36	5	0.139
3	50	0	0.000
4	47	5	0.106
5	48	0	0.000
6	54	3	0.0555
7	50	0	0.000
8	42	1	0.0239
9	32	5	0.156
10	40	2	0.050
11	47	2	0.0425
12	47	4	0.085
13	46	1	0.0217
14	46	0	0.000
15	48	3	0.00625
16	39	0	0.000
Total	720	36	

Solution:

The average sample size

$$= \frac{720}{16} = 45$$

The average fraction defectives

$$\bar{p} = \frac{\text{Total defectives in all samples}}{\text{Total inspected in all samples}} = \frac{36}{720} = 0.05$$

$$UCL_p = \bar{p} + 3\sqrt{\bar{p}(1-\bar{p})/n}$$

$$= 0.05 + 3\sqrt{\frac{0.05(1-0.05)}{45}}$$

$$= 0.14747$$

$$LCL_p = 0.05 - 0.09747$$

$$= -0.04747 = 0$$

Reading number 9 goes out of control. Therefore, the process does not exhibit statistical control.

Problem 3:

A manufacturer purchases small bolts in cartons that usually contain several thousand bolts. Each shipment consists of a number of cartons. As a part of the acceptance procedure for these bolts, 400 bolts are selected at random from each carton and are subjected to visual inspection for certain defects. In a shipment of 10 cartons the respective percentage of defectives in the samples from each carton are 0, 0, 0.5, 0.75, 0, 2.0, 0.25, 0, 0.25 and 1.25. Does this shipment of bolts appear to exhibit statistical control with respect to the quality characteristics examined in the inspection?

Solution:

Average fraction defective

$$\bar{p} = \frac{Total\ number\ of\ defectives}{Total\ number\ inspected}$$

Therefore $\bar{p}$

$$= \frac{(0+0+0.5+0.75+0+2.0+0.25+0+0.25+1.25)\times\frac{400}{100}}{400\times 10} = 0.005$$

$$UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.005 + 3\sqrt{\frac{0.005\times 0.995}{400}}$$

$$= 0.015580$$

$$LCL_p = 0.005 - 3\sqrt{\frac{0.005 \times 0.995}{400}}$$

$$= - 0.00558 = 0$$

Since it is never possible to obtain a negative population of defectives, the lower control limit is taken as zero.

After comparing the reading with UCL_p *and* LCL_p it is found that reading number $6 = 2 \times \frac{1}{100} = 0.02$ falls outside the upper control limit. Hence the shipment does not exhibit statistical control.

Exercises

1. In a manufacturing process, the number of defectives found in the inspection of 15 lots of 400 items each are given below:

Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
No. of defectives	2	5	0	14	3	0	1	0	18	8	6	0	3	0	6

   (a) Determine the trial control limits for np chart and state whether the process is in control.
   (b) What will be new value of mean fraction defective if some obvious points outside control limit are eliminated? What will be the corresponding upper and lower control limits? Examine whether the process is still in control or not.
2. The following table gives the number of defectives for lot number ten goes out of control.

Air plane Number	**Number of missing rivets**	**Air plane Number**	**Number of missing rivets**	**Air plane Number**	**Number of missing rivets**
1	8	10	12	19	11
2	16	11	23	20	9
3	14	12	16	21	10
4	19	13	9	22	22
5	11	14	25	23	7
6	15	15	15	24	28
7	8	16	9	25	9
8	11	17	9		
9	21	18	14		

Find $\bar{c}$ compute trail control limits and plot control chart for c. What values of c would you suggest for the subsequent period?

3. What is difference between a defect and defective?
4. Outline the theory underlying control chart for defects.

5. How will you classified defects? Explain.
6. How do you compare p chart with $\overline{X}$ and R chart?
7. Write short notes on:
   (i). Purpose of p chart
   (ii). Basis for control limits on c chart
   (iii). Classification of defects
   (iv). Comparison between attribute control charts and variable control charts.
8. Explain the construction and operations of np chart and p chart.
9. Explain the concept of c chart and u chart.

Operating Characteristic curve for Control Charts (OC Curve)

Control charts are used to detect the shifts in advance. OC curves for control charts will help to detect the shifts.

Consider sample averages $\bar{x}_1, \bar{x}_2, .. \bar{x}_n$ and their common average is taken as $\mu_0 = \bar{x}$. We know that, standard error of $\bar{x} = \frac{\sigma}{\sqrt{n}}$. It is also noted that the distance between central line at $\mu_0 = \bar{x}$ and any control limit is given by $L\frac{\sigma}{\sqrt{n}}$

That is,

$$UCL = \mu_0 + L\frac{\sigma}{\sqrt{n}}$$

$$LCL = \mu_0 - L\frac{\sigma}{\sqrt{n}}$$

$$CL = L\frac{\sigma}{\sqrt{n}}.$$

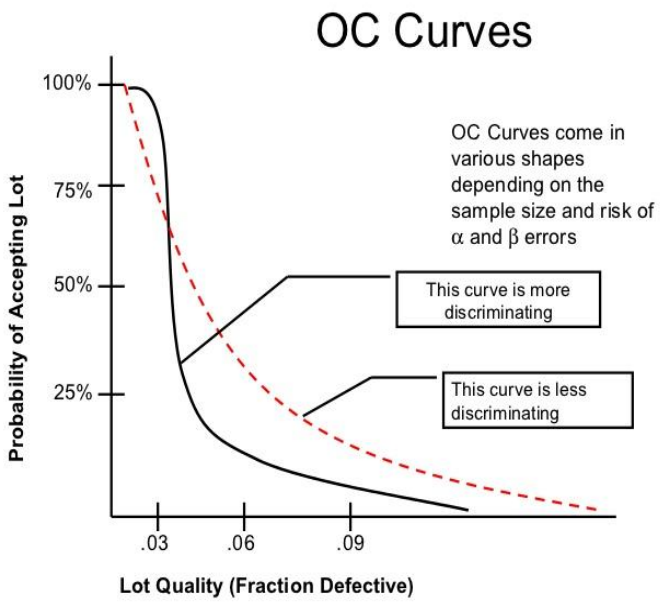

Figure: OC Curve for Control Chart

Let μ_1 be a point which falls outside the control limit. This point is to be detected by using OC function. Now we define $\mu_1 = \mu_0 + k\sigma$.

Let β be the probability of not detecting the shifts.

That is, $$\beta = P\left[LCL \le \bar{x} \le UCL\right]$$

$$= P\left[\frac{LCL - \mu_1}{\sigma/\sqrt{n}} \le \bar{x} \le \frac{UCL - \mu_1}{\sigma/\sqrt{n}}\right]$$

$$\beta = F\left(L - k\sqrt{n}\right) - F\left(-L - k\sqrt{n}\right)$$

Here $1 - \beta$ is the probability of detective the shifts and its reciprocal is called average run length

$$ARL = \frac{1}{1-\beta}.$$

Modified Control Limits (Rejection Control Limits)

If any control chart 3σ limit plays a vital role. If μ and σ are process mean and process standard deviation respectively, then the limits $\mu \pm 3\sigma$ are called natural tolerance limits. The overall width 6σ is named as natural tolerance which means that if any value falls outside 6σ level, assignable causes are present then the process may be rejected or revised. Consider the maximum and minimum values from the given observations and X_{max} and X_{min} denote USL and LSL respectively for some quality characteristic. The relationship between natural tolerance limits and specification limits are given below.

i) USL – LSL > 6σ

ii) USL – LSL ≈ 6σ

iii) USL – LSL < 6σ

iv) Among these three relations, if the first relation USL – LSL > 6σ holds good, then modified control limits exhibit the relation between specification limits and the values of $\bar{x}$ in average chart.

If the universe is at the highest accepting position, the process average (central line) will be at a distance 3σ below USL and when the universe is at its lowest accepting position, the process average is at a distance 3σ above LSL.

Similarly the distance between central line and upper control limit (or) lower control limit is $\frac{3\sigma}{\sqrt{n}}$.

For detailed study we have to form a central band instead of fixing central line at $\bar{\bar{x}}$. The upper and lower edges of the central band are defined as

Upper edge, UE = USL – 3σ

Lower edge, LE = LSL + 3σ

The below chart reduced that the highest and lowest value of upper control limit and lower control limit are identical with upper rejection limit and lower rejection limit respectively. On the basic of the diagram, rejection limits are defined as

$$URL = UE + \frac{3\sigma}{\sqrt{n}}$$

$$LRL = LE - \frac{3\sigma}{\sqrt{n}}$$

These equations are rewritten by using the expressions for UE and LE are follows

$$URL = UCL - 3\sigma + \frac{3\sigma}{\sqrt{n}}$$

$$LRL = LCL + 3\sigma - \frac{3\sigma}{\sqrt{n}}$$

These rejection limits are called modified control limits.

Applications of c-chart

The universal nature of Poisson distribution as the law of small numbers makes the c-chart technique quite useful. In spite of the limited field of application of c-chart (as compared to $\bar{X}$, R and p charts), there do exist situation in industry where c-chart is definitely needed. Some of the representative types of defects to which c-chart can applied with advantages are:

1. c is number of imperfections observed in a bale of cloth.
2. c is the number of surface defects observed in (i) roll of coated paper or a sheet of photographic film and (ii) a galvanised sheet or a painted, plated or enamelled surface of given area.
3. c is the number of defects of all types observed in aircraft sub-assemblies or final assembly.
4. c is the number of breakdowns at weak sports in insulation in a given length of insulated wire subject to a specified test voltage.
5. c is the number of defects observed in stains or blemishes on a surface.
6. c is the number of soiled packages in a given consignment.
7. c-chart has been applied to sampling acceptance procedures based on number of defects per unit, example, in case of inspection of fairly complex assembled units such as T.V. sets, aircraft engines, tanks, machine-guns, etc., in which there are very many opportunities for the occurrence of defects of various types and the total number of defects of all types found by inspection is recorded for each unit.
8. c-chart technique can be used with advantages in various fields other than industrial quality control, example, it has been applied (i) to accident statistics (both of industrial accidents and highway accidents), (ii) in chemical laboratories and (iii) in epidemiology.

Unit III

Acceptance Sampling

3.1. Introduction

3.2. Sampling Inspection

3.3. Basic Definitions in Acceptance Sampling

3.4. Rectifying Inspection Plan

3.5. Average Outgoing Quality Limit (AOQL)

3.6. Average Sample Number (ASN) and Average Total Inspection (ATI)

3.1. Introduction

Acceptance sampling is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot as either conforming or not conforming to quality specification.

Inspection for acceptance purpose is carried out at many stages in manufacturing. There are generally two ways in which inspection is carried out: (i) 100% inspection (ii) sampling inspection

In 100% inspection all the parts or products are subjected to inspection, whereas in sampling inspection only a sample is drawn from the lot and inspected.

A sample may be defined as the number of times drawn from a lot, batch or population for inspection purpose.

3.2. Sampling Inspection

Sampling inspection can be defined as a technique to determine the acceptance or rejection of a lot or population on the basis of number of defective parts found in a random sample drawn from the lot. If the number of defective items does not exceed a predefined level, the lot is accepted, otherwise it is rejected.

Sampling inspection is not a new concept. In our daily life we use sampling inspection in selecting certain consumable items. For example, while purchasing our annual or monthly requirements of wheat, rice or such other food grains we naturally take a handful of grains to judge its quality for taking purchasing decision. If we are not satisfied we take another sample and after two or three samples from the same or different sources we take purchasing decision. Let us take another example, suppose we want to purchase mangoes we normally take one or two mangoes from the lot and taste its quality, if the samples taken are found good we decide to purchase the required quantity.

Similarly, in engineering sampling inspection is preferred because it is more practical, quick and economical as compared to 100% inspection. The main purpose of acceptance

sampling is to distinguish between good lots and bad lots, and to classify the lots according to their acceptability or non-acceptability.

Advantages of Sampling Inspection

The advantages of sampling inspection are as follows:

1. The items which are subjected to destructive test must be inspected by sampling inspection only.
2. The cost and time required for sampling inspection is quite less as compared to 100% inspection.
3. Problem inspection fatigue which occurs in 100% inspection is eliminated.
4. Smaller inspection staff is necessary.
5. Less damage to products because only few items are subjected to handling during inspection.
6. The problem of monotony and inspector error introduced by 100% inspection is minimised.
7. The most important advantage of sampling inspection is that, it exerts more effective pressure on quality improvement. Since the rejection of entire lot on the basis of sampling brings much stronger pressure on quality improvement than the rejection of individual articles.

Limitations of Sampling Inspection

1. Risk of making wrong decisions

   However, in sampling inspection, since only a part is inspected, it is inevitable that the sample may not always represent the exact picture obtaining in the lot and hence, there will be likelihood or risk of making wrong decisions about the lot. This wrong decision can be made in two ways. Firstly, a really good lot (that is, containing less proportion of defective than specified) may be rejected because the sample drawn may be bad. Secondly, a really bad lot (that is, a lot containing greater proportion of defectives than specified) may be accepted because the sample drawn may be good. In the former case, the producer has to suffer a risk of his good lots being rejected and hence the associated risk (chance) is called as the producer's risk. In the letter case, the consumer runs the risk of accepting bad lots and hence the associated risk is called as consumer's risk.
2. The sample usually provides less information about the product than 100 per cent inspection.
3. Some extra planning and documentation is necessary.

   However, in scientific sampling plans, these risks are quantified and the sampling criteria are adjusted to balance these risks, in the light of the economic factors involved.

   The success of a sampling scheme depends upon the following factors:

   (i) Randomness of samples
   (ii) Sample size
   (iii) Quality characteristic to be tested

(iv) Acceptance criteria
(v) Lot size

Industrial Uses of Acceptance sampling

1. To determine the quality and acceptability of incoming raw materials, component parts, products etc.
2. To decide the acceptability of semi finished products for further processing as it undergoes the operations from machine to machine or section to section within the factory.
3. To determine the quality of outgoing products.
4. For improving maintaining and controlling the quality of the products manufactured.

3.3. Basic in Acceptance Sampling

In any production process, the producer gets his lot checked at various stages or the customer is anxious to satisfy himself about the quality of goods. An ideal way of doing this, seems to inspect each and every item presented for acceptance. 100 percent inspection should be taken under the following circumstance:

1. The occurrence of a defect may cause less of life or serious causality.
2. A defect may cause serious malfunction of equipments.
3. If testing is destructive like crackers, bulbs, shells etc., it is absolutely nonsensical to talk of hundred percent inspection.

From practical and economic considerations, sampling procedures are adopted such as a lot is accepted or rejected on the basic of samples drawn at random from the lot.

It is noted that, if a scientifically designed sampling inspection plan is used it provides sufficient protection to both producer and consumer. The main objective of infection is to control the quality of product. Sampling inspection ensures that, the quality lot is accepted according to the specifications of the consumer.

The guide lines of a sampling procedure are given below

1. It should give a definite assurance against a passing unsatisfactory lot.
2. The inspection expenses should below as possible.

1. Acceptance Quality Level (AQL)

If a lot has small fraction defective, we do not wish to reject and considered as a good lot. Let p_1 denotes a lot of quality. For small fraction defective,

$$P_r \text{ (Rejecting a lot of quality } p_1) = 0.05$$

This implies that,

$$P \text{ (Accepting a lot of quality } p_1) = 0.95$$

Here, p_1 is known as acceptance quality level and the quality of this lot is satisfactory by the consumers.

2. Lot Tolerance Percent Defective (LTPD) or Rejecting Quality Level (RQL)

The consumer is not willing to accept the lot having proportion defective p_1 greater. $100 \times p_1$ is called lot tolerance percent defective. This is the quality level in which consumer regards as rejectable and it is usually named as rejecting quality level.

A lot of quality p_1 is to be accepted at some arbitrary and small fraction of time, usually 10 percent.

3. Process Average Fraction Defective $(\bar{p})$

In any production process, the quality of a product tends to settle down to some desired level which may be expected to be more or less the same for every day for a particular machine. If this level could be maintained and if the process is working free from assignable causes of variation, the inspection could often be dispensed with. But in practice as a result of failure of machine and laziness of workers, the quality of the product may suddenly decrease.

The process average of any manufactured product is obtained by computing the percentage of defectives in the product over a long time and it is denoted by $\bar{p}$.

4. Producer's Risk (α)

The producer has to face the situation that some good lots will be rejected. The producer might demand sufficient protection against such contingencies happening frequently just as the consumer can claim reasonable protection against accepting too many bad lots. The probability of rejecting a lot with $100\,\bar{p}$ as the process average percent defective is called producers risk (P_p). This probability is denoted by α.

$$\alpha = (P_p) = P \text{ (Rejecting a lot of quality } \bar{p}\text{)}$$

This is also known as Type I error.

5. Consumer's Risk (β)

In some critical situations consumer has to face the problem that he may accept certain percentage of undesirable bad lots, which have lots of quality p_1 or greater fraction defective. The probability of accepting a lot with fraction defective p_1 is denoted by β. Usually consumer's risk is taken at 10% level.

$$Pc = P \text{ (Accepting a lot of quality } p_1\text{)} = \beta$$

This is also known as Type II error.

3.4. Rectifying Inspection Plans

The inspection of the rejected lots and replacing the defective pieces by standard pieces. This process eliminates the number of defectives in the lot and hence to improve the

quality of the product. These plans are called rectifying inspection plan and it was first introduced by Dodge and Roming before World War II. This plans enable the manufacture to have an idea about the average quality of the product. The rectification process is performed in different stages of production through the combination of production, the sampling inspection and rectification of rejected lots. Most of the rectifying inspection plans for lot by lot sampling call for 100% inspection of the rejected lots and replacing the defective pieces by good once.

The two important quality control concepts related to rectifying control concepts related to rectifying inspection plans. They are Average Outgoing Quality (AOQ) and Average Total Inspection (ATI).

(i) Average quality of the product after sampling and 100% inspection of rejected lots is called AOQ.

(ii) The average amount of inspection required for the rectifying inspection plan is called ATI.

3.5. Average Outgoing Quality Limit (AOQL)

Sometimes the consumer is guaranteed a certain quality level after inspection - regardless of what quality level is being maintained by the producer. Let the producer's fraction defective, i.e., lot quality before inspection be 'p'. This is termed as 'incoming quality'. The fraction defective of the lot after inspection is known as 'outgoing quality' of the lot. The expected fraction defective remaining in the lot after the application of the sampling inspection plans is termed as AOQ $\tilde{p}$. Obviously, it is a function of the incoming quality 'p'.

Remark

For rectifying inspection single sampling plan calling for 100% inspection of the rejected lots, the AOQ values are given by the formula:

$$\tilde{p} = AOQ = \frac{p(N-n)P_a}{N} \tag{1}$$

Where N is lot size, n is sample size and P_a is the probability of acceptance of the lot.

If n is small compared with N, then a good approximation of the outgoing quality is given by,

$$\tilde{p} = AOQ = p\,P_a \tag{2}$$

If the defective pieces found are not repaired or replaced, then the formula must be modified to

$$AOQ = \frac{p(N-n)P_a}{N - np - p(1-P_a)(N-n)} = \frac{p(N-n)P_a}{N - p[nP_a + N(1-P_a)]} \tag{3}$$

This formula is not generally used and if p is small, there is not much difference between equation (2) and (3).

In general, if p is the incoming quality and a rectifying inspection plan calling for 100% inspection of the rejected lots is used, then the AOQ of the lot will be given by,

$$AOQ = p\,P_a(p) + 0 \times [1 - P_a(p)] = p\,P_a(p) \qquad (4)$$

Because

(i) $P_a(p)$ is the probability of accepting the lot of quality 'p' and when the lot is accepted on the basis of the inspection plan, the outgoing quality of the lot will be approximately same as the incoming lot quality 'p'.

(ii) $1 - P_a(p)$ is the probability of rejection of the lot is rejected after sampling inspection and is subjected to 100% screening and rectification, the AOQ is zero.

For a given sampling plan, the value of AOQ can be plotted for different values of p to obtain the AOQ curve.

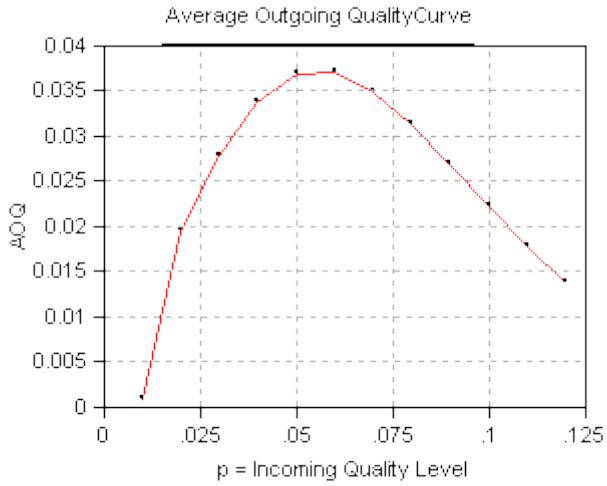

Figure: Average Outgoing Quality Curve

3.6. Average Sample Number (ASN) and Average Total Inspection (ATI)

The average sample number (ASN) is the expected value of the size required to take a decision about the acceptance or rejection of the lot. Suppose the lot is accepted, ASN is equal to the sample size. It is also noted that ASN is a function of incoming lot quality p.

The expected number of items inspected per lot is to arrive a decision as acceptance or rejection or rectification after 100% inspection is called ATI.

ATI is the function of ASN and average size of inspection of remaining units in the lot.

ATI = ASN + (Average size of inspection in the remaining units) (1)

If the lot is accepted on the basis of sampling inspection plan, ATI = ASN. If the lot is rejected, ATI > ASN.

For example, if a single sampling plan number of inspected item in the lot is equal to sample size, if the lot is accepted. i.e. ASN = n. In a single sampling plan after inspecting the

sample, the lot is submitted for 100% inspection, the inspected items vary from lot to lot. As we already told, if the lot is equal to sample size (n). If the lot is rejected, the number of inspected items is lot size (N).

Let P_a be the probability of acceptance and 1-P_a be the probability of rejection. As per the above statements of acceptance and rejection, ATI is defined as,

i.e. Sample size and rejection of the lot.

$$ATI = nP_a + N(1-P_a)$$

$$= nP_a + (N+n-n)(1-P_a)$$

$$= nP_a + P_a(N-n)(1-P_a) + n(1-P_a)$$

$$ATI = n = (N-n)(1-P_a)$$

As in the case of OC curves, fixing the fraction defectives and find the probabilities of acceptance. Then calculate the values of ATI by using the above relation by taking the fraction defective along x axis and ATI along y axis and get the required ATI curve. The shape of the ATI curve is the mirror image of OC curve.

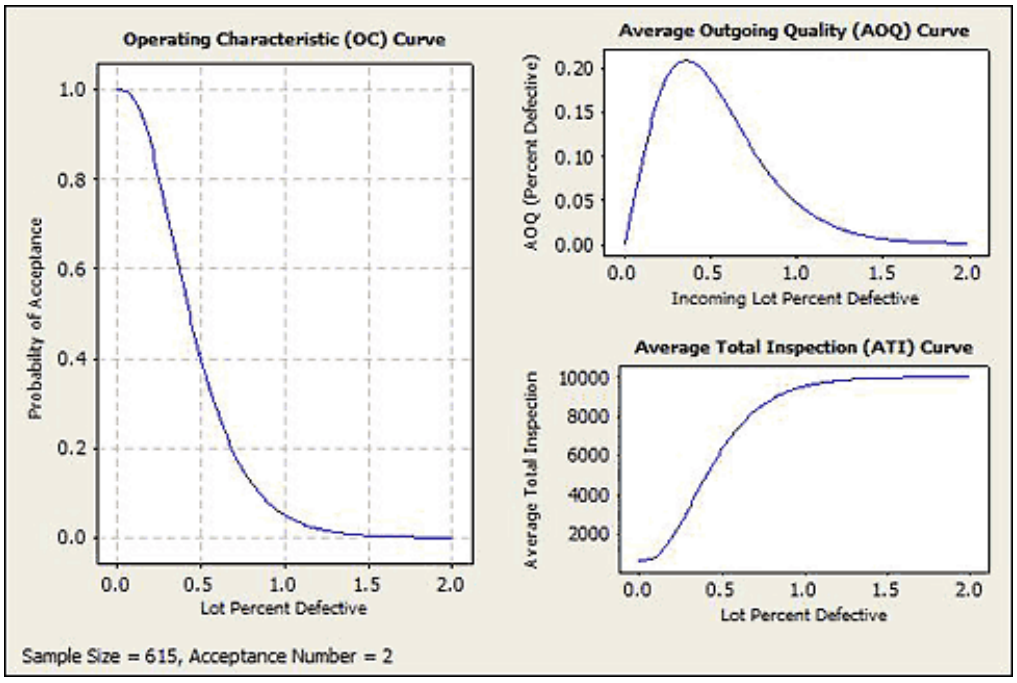

Figure: Operating Characteristic Curve, Average Outgoing Quality Curve and Average Total Inspection Curves

Exercises

1. What is meant by sampling inspection by attributes?
2. What is an acceptance sampling?
3. Define producer's and consumer's risks on operating characteristic curve.
4. Discuss the following:

   (i) AOQ and AOQL (ii) Types of OC curves (iii) AQL and LTPD

Unit IV

Acceptance Sampling by Attributes

4.1. Introduction

4.2. Single Sampling Plan

4.3. OC, AOQ, ATI and ASN for Single Sampling Plan

4.4. Double Sampling Plan

4.5. OC, ASN and ATI for Double Sampling Plan

4.6. Comparison of Single sampling and Double sampling plans

4.1. Introduction

In attributes sampling, a predetermined number of units from each lot is inspected. Each unit is graded as conforming or nonconforming. A nonconforming unit is defined as a unit that does not meet product specifications for one or more quality characteristics. If the number of nonconforming units is less than the prescribed minimum, the lot is accepted; otherwise, the lot is rejected. There are several types of plans for attributes sampling. Four of these are single sampling, double sampling, multiple sampling and sequential sampling plans. These are discussed in the following sections.

4.2. Single Sampling Plan

In any production process 100% inspection requires more time, more inspection cost and soon. For reducing time and cost, we may apply sampling procedure. Dodge and Roming have established single sampling plan, double sampling plan, multiple sampling plan, etc.

The working principle of single sampling plan is described as follows:

A Single Sampling Plan is denoted as $\begin{pmatrix} N \\ n \\ c \end{pmatrix}$. Let N be the lot size, n be the sample size and c be the acceptance number. The acceptance number is otherwise known as maximum allowable number of defectives in the sample.

Select a random sample of size n from a lot of size N.

1. Inspect all the articles in the sample.
2. The number of defectives in the sample be ‘d’.
3. If $d \leq c$, accept the lot, replacing defective pieces found in the sample by non-defective (standard) pieces.
4. If $d > c$, reject the lot and we inspect the entire lot and replace all the defective pieces by standard pieces.

The working principle of Single Sampling Plan is exhibited in the following chart:

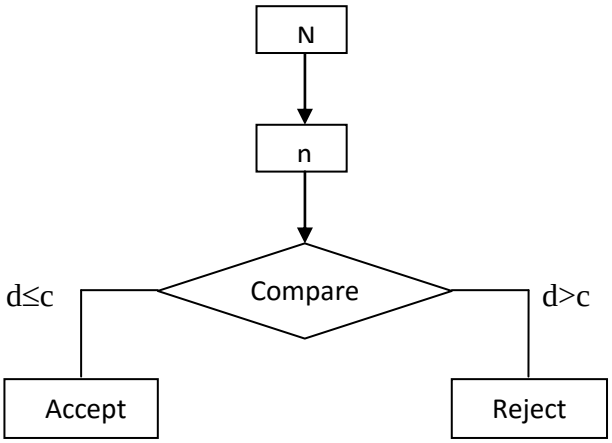

Figure: Single Sampling Plan

For analysing the quality of the product, quality control engineers are interested to design Operating Characteristic (OC) curve, Average Outgoing Quality (AOQ) curve, Average Sample Number (ASN) curve and Average Total Inspection (ATI) curve.

4.3. OC, AOQ, ATI and ASN for Single Sampling Plan

In acceptance sampling plan, there are many curves to detect the process variation and find out probability of acceptance, probability of rejection, ASN and so on. Among the curves, OC curve is a simple and basis of either fraction defective p or number of defectives in sample (np) verses probability of acceptance.

One may able to construct OC curve for single sampling plan with finite or infinite size. OC curve is classified into two types such as type A and type B OC curves. The construction of OC curves for single sampling plan is described as follows:

Consider a single sampling plan $\begin{pmatrix} N \\ n \\ c \end{pmatrix}$. Let p be the fraction defective gives Np is the defective in lot and x is the number of defectives in sample.

According to the principle of hypergeometric distribution, probability of x defectives with fraction defective p is given by

$$g(x, p) = \frac{\begin{pmatrix} Np \\ x \end{pmatrix}\begin{pmatrix} N - Np \\ n - x \end{pmatrix}}{\begin{pmatrix} N \\ n \end{pmatrix}} \qquad (1)$$

Probability of acceptance of the given lot is defined as the cumulative probabilities of x = 0, 1, …, c defectives. By using equation (1) we have

$$L(p)=P_a=\sum_{x=0}^{c} g(x,p)=\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{2}$$

Suppose p is very small, the RHS of equation (2) it is approximately equal to binomial expression of $\left[\left(1-\frac{n}{N}\right)+\frac{n}{N}\right]^{Np}$. When apply binomial property to the equation (2), it reduces to

$$L(p)=\sum_{x=0}^{c}\binom{Np}{x}\left(\frac{n}{N}\right)^{x}\left(1-\frac{n}{N}\right)^{Np-x} \tag{3}$$

In equation (3), we considered $\frac{n}{N}$ is very small and Np is large ($Np\to\infty$). Then the equation (3) becomes

$$L(p)=\sum_{x=0}^{c} e^{-np}\frac{(np)^{x}}{x!} \tag{4}$$

By using different values of any p and c, we get different cumulative probabilities from cumulative Poisson distribution tables. On taking 'np' along x axis and L(p) along y axis and draw the required OC curve.

Determine the parameters of Single Sampling Plan: (Determine n and C)

Consider a single sampling plan $\binom{N}{n}{c}$. Here N be the lot size, n be the sample size and c be the acceptance number. In any single sampling plan the lot size N either finite or infinite and always assume that it is a known quantity. But n and c are generally unknown quantities and they are to be determined.

Let us assume p be the fraction defective or a lot of incoming quality. Out of N units, the number of defective pieces is taken as 'Np' and number of non-defective pieces is 'N-Np'. Suppose x is the number of defectives in the sample then find out the probability distribution function of x defectives in the lot. The number of ways of selecting x defectives from Np defectives is $\binom{Np}{x}$. Similarly the number of ways of selecting (n-x) non-defectives from, the number of non-defectives in the lot is $\binom{N-Np}{n-x}$.

The number of ways of selecting n units from N units is $\binom{N}{n}$.

From the above statement, the probability of getting x defectives is given by

$$g(x,p)=\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{1}$$

This is the probability mass function of hypergeometric distribution.

The probability of accepting a lot with fraction defective p is obtained by taking the summation x=0, 1, …, c in equation (1) and get

$$P_a=\sum_{x=0}^{c} g(x,p)=\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{2}$$

This equation (2) is also known as consumer's risk.

$$\text{i.e. } P_c=\sum_{x=0}^{c} g(x,p)=\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{3}$$

On the other hand producer's risk is given by

$$P_p=1-P_c=1-\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}} \tag{4}$$

Equations (3) and (4) are enough to find the values of n and c, but it is cumbersome to solve the equations. So we reduce equation (3) and (4) in the form of the probability mass function of binomial distribution.

When p is very small the right hand side of equation (3) is approximately equal to the first c+1 terms of binomial expansion $\left[\left(1-\frac{n}{N}\right)+\frac{n}{N}\right]^{Np}$

Then equation (3) becomes

$$P_c=\sum\binom{Np}{x}\left(\frac{n}{N}\right)^{x}\left(1-\frac{n}{N}\right)^{Np-x} \tag{5}$$

In this stage we considered $\frac{n}{N}$ is very small, Np is very large and 'np' is taken as Poisson variate and also taking the limit as $Np\to\infty$, then equation (5) reduces to

$$P_c = \sum_{x=0}^{c} e^{-np} \frac{(np)^x}{x!} \tag{6}$$

Similarly equation (4) reduces to

$$P_p = 1 - P_c = 1 - \sum_{x=0}^{c} e^{-np} \frac{(np)^x}{x!} \tag{7}$$

On solving equation (6) and (7) we get a required values of the parameter n and c.

4.4. Double Sampling Plan

Dodge and Roming have proposed another sampling scheme which is called second sampling method or double sampling procedure. In this method, a second sampling permitted if the first samples fails. If we are not able to take decision based on first sample, then draw second sample and conclude decision on the basis of second sample. The double sampling plan is denoted as $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. Here N is the lot size, n_1 is the first sample size. c_1 is the acceptance number for first sample or maximum allowable number of defectives in the first sample. n_2 is the second sample size. c_2 is the acceptance number for both samples combined. d_1 is the number of defective items in the first sample, d_2 is the number of defective in the second sample.

The working principle of double sampling plan is described as follows

1. Consider a lot of size N from a production plan.
2. Take a sample of size n_1 from the lot.
3. Inspect the first sample and find out the number of defective items, let it be 'd_1'.
4. If $d_1 \le c_1$, accept the lot and replace the bad items by standard items.
5. If $d_1 > c_2$, reject the lot and apply 100% inspection in the lot. Remove all bad items and include standard ones to adjust the lot size as N.
6. If $c_1 + 1 < d_1 \le c_2$ then take a second sample of size n_2 from the remaining lot.
7. If $d_1 + d_2 \le c_2$ accept the lot and replace the bad items by standard items.
8. If $d_1 + d_2 > c_2$ reject the whole lot and apply 100% inspection.

Replace defective items by standard items. This working principle of double sampling plan is exhibited in the following chart:

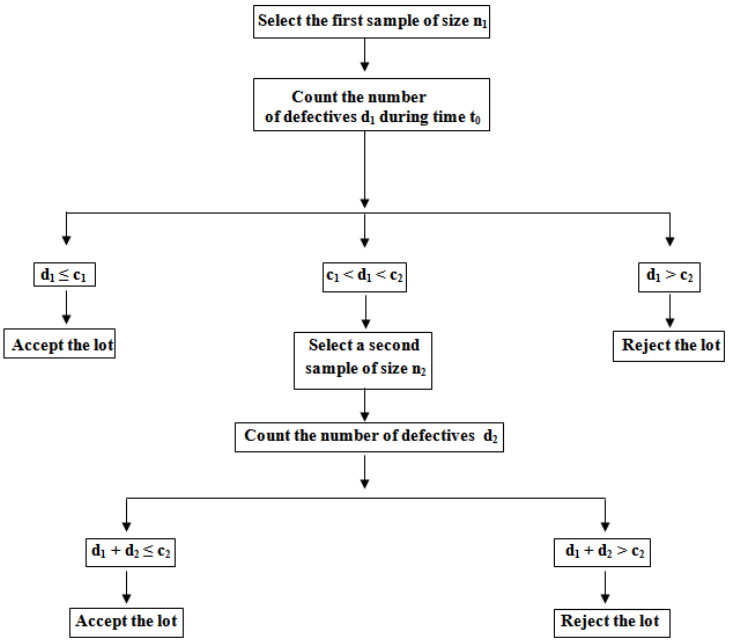

Figure: Double Sampling Plan

4.5. OC, ASN and ATI for Double Sampling Plan

Consider a double sampling plan $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. Here N is the lot size. n_1 *and* n_2 are fixed first and second sample sizes respectively, c_1 *and* c_2 are acceptance numbers corresponding to first and both combined samples respectively. The construction of OC curve for double sampling plan is very cumbersome, since the selection of second sample depends upon the number of nature of first sample. Let x be the number of defectives in the first sample, if x lies between (0, c_1), then we accept the lot and find the probability of acceptance. This implies that there is no chance to select second sample. We know that the probability of getting x defective with fraction defective p is g(x, p). The number of defectives 'x' follows hypergeometric distribution and its probability mass function is given by,

$$g(x,p)=\frac{\begin{pmatrix} Np \\ x \end{pmatrix}\begin{pmatrix} N-Np \\ n_1-x \end{pmatrix}}{\begin{pmatrix} N \\ n_1 \end{pmatrix}} \qquad (1)$$

The cumulative probability of x defectives (x = 0,1,2,…,c_1) is called probability of acceptance of first sample. Summing the equation (1), by apply x = 0,1,2,…,c_1 and get the probability of acceptance of first sample.

$$P_{a1}=\sum_{x=0}^{c_1} g(x,p)$$

$$= \sum_{x=0}^{c_1} \frac{\binom{Np}{x}\binom{N-Np}{n_1-x}}{\binom{N}{n_1}} \qquad (2)$$

Suppose number of defectives (x) in the first sample satisfy the inequality $c_1 + 1 \le x \le c_2$ be selected second sample of size n_2. After inspecting the second sample and observed that there are 'y' defectives. In this stage, we have to construct the conditional probability of 'y' defectives in the second sample when there are x defectives in the first sample. Let it be h(y, p/x). The number of ways of selecting y defectives from Np-x defectives is $\binom{Np-x}{y}$.

In the second sample, there are $n_2 - y$ non-defectives and $N - n_1 - (Np - x)$ non-defectives are available in the lot. The number of ways of selecting $n_2 - y$ non-defectives from $[N - n_1 - (Np - x)]$ non-defectives is $\binom{(N-n_1)-(Np-x)}{n_2-y}$.

Finally the sample of size n_2 is selected from the remaining lot $N - n_1$. In this case, the number of ways selecting n_2 units is $\binom{N-n_1}{n_2}$. By using the above statements, the conditional probability of 'y' defectives in the second sample when there are x defectives in the first sample is obtained as

$$h(y, p/x) = \frac{\binom{Np-x}{y}\binom{N-n_1-(Np-x)}{n_2-y}}{\binom{N-n_1}{n_2}} \qquad (3)$$

Let P_{a2} be the probability of acceptance in the second sample under the condition that 'x' defectives in the first sample and y defectives in the second sample based on the non-acceptance if the first sample with x defectives,

$$P_{a2} = \sum\sum g(x, p) \times h(y, p/x) \qquad (4)$$

By using the equations (1), (2) in equation (4),

We get

$$P_{a2} = \sum_{y=0}^{c_2-x} \sum_{x=c_1+1}^{c_2} \frac{\binom{Np-x}{x}\binom{N-Np}{n_1-x}}{\binom{N}{n_1}} \frac{\binom{Np-x}{y}\binom{N-n_1-(Np-x}{n_2-y}}{\binom{N-n_1}{n_2}} \qquad (5)$$

By adding equation (2) and (5), we get the probability of acceptance P_a in double sampling plan as

$$P_a = P_{a1}+P_{a2} \quad (6)$$

For different values of lot quantity (fraction defective) 'p' we compute corresponding probabilities P_a and by considering p and P_a, we can draw the OC curve for the given double sampling plan.

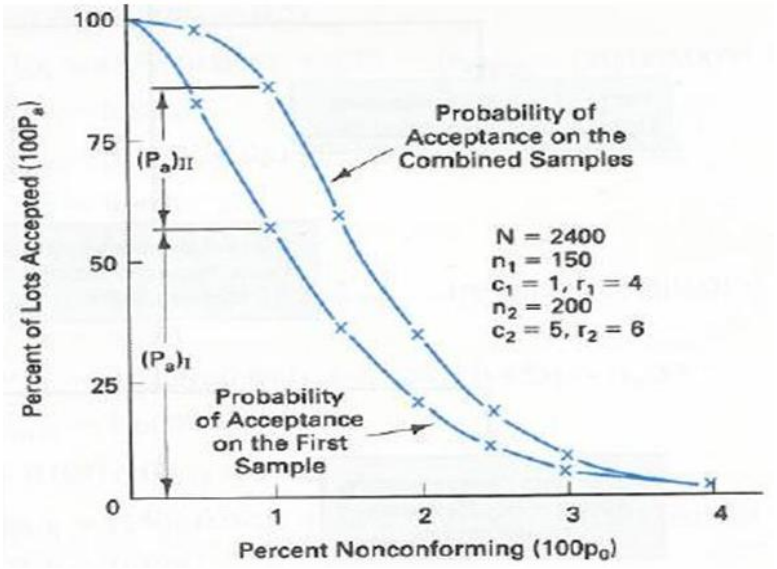

Figure: OC Curve for Double Sampling Plan

4.6. Comparation of Single Sampling and Double Sampling Plans

1. Single sampling plan are simple, easy to design and administer.
2. An advantage of double sampling plan over single sampling plan seems to be psychological.
3. Leyman can feel that reject a lot on the basis of single sampling plan is undesirable but they are satisfied when the lot is rejecting after inspecting the second sample.
4. Double sampling plan involves on the average less amount of inspection than that of single sampling plan for the same quality assurance.
5. Under double sampling plan, good quality lot will be accepted and bad quality lot will be rejected on the basis of first sample.
6. When a lot is rejected on the basis of second sample, without completely inspecting the entire lot, double sampling plan requires 25% or 33% less inspection than single sampling plan.
7. Unit cost of inspection for double sampling plan may be higher than that for single sampling plan.
8. OC curves for double sampling plan are steeper than OC curves for single sampling plan. This implies that the discriminant power of double sampling plan is higher than that of single sampling plan.

Also, we compare SSP and DSP in the following ways:

	Single Sampling Plan	Double Sampling plan
1. Average number of pieces inspected per lot	Largest	In between single and multiple plans
2. Cost of administration	Lowest	In between single and multiple plans
3. Information available regarding prevailing quality level	Largest	In between single and multiple plans
4. Acceptability to producers	Less (gives only one change of passing the lot)	Most acceptable

Types of OC curves

Dodge and Roming have classified the OC curves into two types such as type A and type B OC curves. Type A OC curve are drawn for the sampling plans with finite lots. The curves give the probability of acceptance of the lot quality. These curves should be drawn by computing probability of acceptance based on hypergeometric distribution or binomial distribution or Poisson distribution. Since the above discrete distributions, they give discontinuous curves. But in practice it is drawn as continuous curves.

Type B OC curves are drawn for the sampling plans which are having infinite lot size. The curves gives the probability of acceptance of product quality. Type B curves are drawn based on the probability of acceptance relative to binomial distribution. The probabilities from binomial distribution give exact OC curve. But the probabilities from Poisson distribution give satisfactory approximation curve.

In general, if the sample size n is not more than one-tenth of the lot size N, type A and type B OC curve may be considered as identical. Type A OC curve may be desirable in evaluating consumer's risk with respect to individual lots. Procedures risk is approximately measured based on type B OC curve.

Type A curve always falls below the type B curve, it follows that the use of type B curves tends to give consumer's risk. That is very high.

Average Outgoing Quality (AOQ) for Single Sampling Plan

Consider a single sampling plan $\begin{pmatrix} N \\ n \\ c \end{pmatrix}$. Average quality of the product after sampling inspection is called AOQ. Take a sample of size n from the lot of size N. Let p be a fraction defective and P_a be the probability of acceptance of the given lot. Similarly $1-P_a$ is the probability of rejection of the given lot. The number of defective units in the sample is consider as 'np'.

After drawing the sample, the remaining units in the lot is N-n. It implies that number of defective units in the remaining lot is (N-n)p out of (N-n) units.

The average of acceptable units of AOQ

$$AOQ = \frac{k(N-n)pP_a}{kN}$$

$$AOQ = \frac{(N-n)pP_a}{N} \text{ If N is finite.}$$

If N is very large $N \to \infty$,

$$AOQ = \frac{(N-n)pP_a}{N}$$

$$= \left(\frac{N}{N} - \frac{n}{N}\right)pP_a$$

$$= \left(1 - \frac{n}{N}\right)pP_a$$

$$AOQ = pP_a$$

If N is infinite. The maximum value of AOQ is called Average Quality Level or maximum available outgoing quality. By taking 'p' along x axis and 'AOQ' along y axis, we get required AOQ curve.

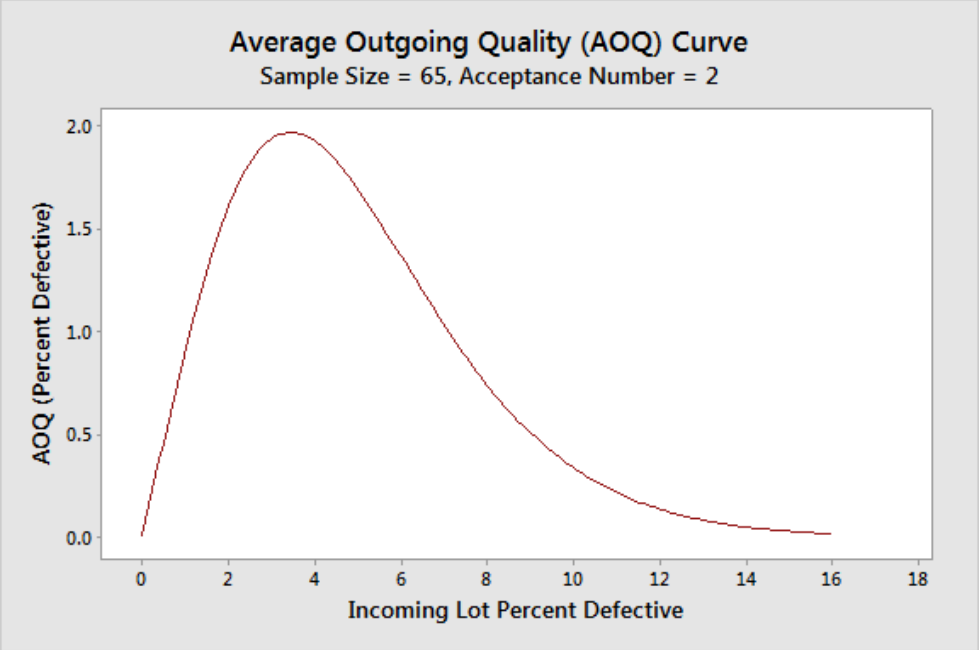

Figure: Average Outgoing Quality for Single Sampling Plan

Average Sample Number (ASN) and Average Amount of Total Inspection (ATI)

The ASN is the expected value of the size required to take a decision about the acceptance or rejection of the lot. Suppose the lot is accepted, ASN is equal to the sample size. It is also noted that ASN is a function of incoming lot quality p.

The expected number of items inspected per lot is to arrive a decision as acceptance or rejection or rectification after 100% inspection is called ATI.

ATI is the function of ASN and average size of inspection of remaining units in the lot.

ATI = ASN + (Average size of inspection in the remaining units).

If the lot is accepted on the basis of sampling inspection plan, ATI = ASN. If the lot is rejected, ATI > ASN.

In single sampling plan, number of inspected item in the lot is equal to sample size, if the lot is accepted. i.e. ASN = n. In a single sampling plan after inspecting the sample, the lot is submitted for 100% inspection, the inspected items vary from lot to lot. As we already told, if the lot is equal to sample size (n). If the lot is rejected, the number of inspected items is lot size (N).

Let P_a be the probability of acceptance and 1-P_a be the probability of rejection. As per the above statements of acceptance and rejection, ATI is defined as,

i.e. Sample size and rejection of the lot.

$$ATI = nP_a + N(1 - P_a)$$

$$= nP_a + (N + n - n)(1 - P_a)$$

$$= nP_a + P_a(N - n)(1 - P_a) + n(1 - P_a)$$

$$ATI = n + (N - n)(1 - P_a)$$

As in the case of OC curves, fixing the fraction defectives and find the probabilities of acceptance. Then calculate the values of ATI by using the above relation by taking the fraction defective along x axis and ATI along y axis and get the required ATI curve. The shape of the ATI curve is the mirror image of OC curve.

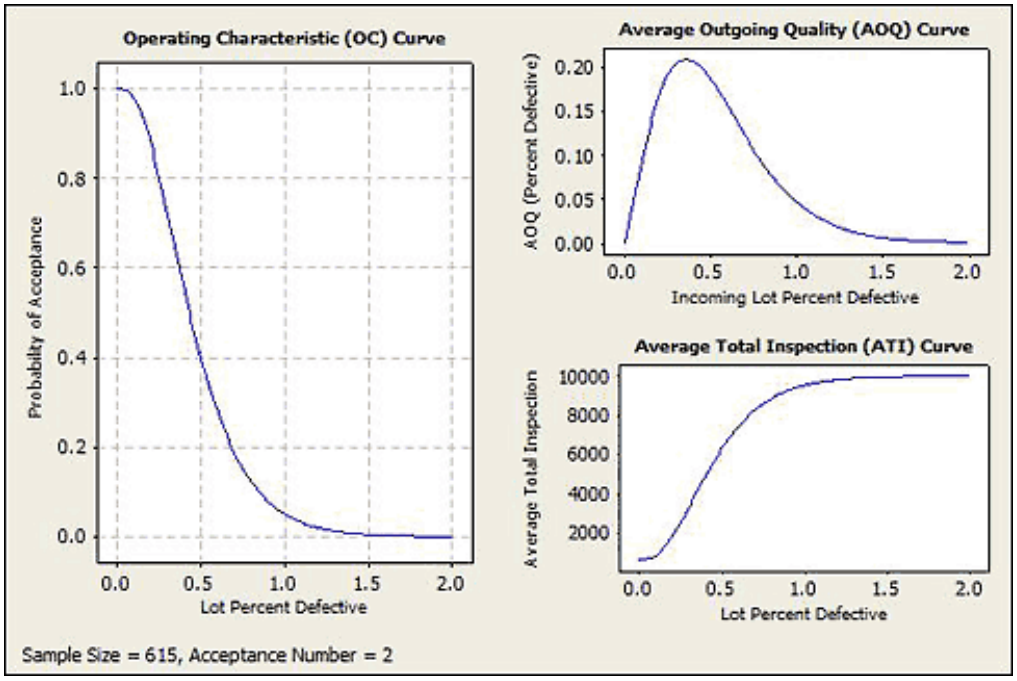

Figure: Operating Characteristic Curve, Average Outgoing Quality Curve and Average Total Inspection Curves for Single Sampling Plan

ASN curve for Double Sampling Plan

Consider a double sampling plan with usual notations as $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. On the basis of inspecting the number of units in any sampling plan, we have to construct the functions of ASN and ATI curves. From a double sampling plan, we select a sample of size n_1 and inspect the units. After inspecting the units we have to take a decision either accept or reject the lot. Let p_1 be the probability of taking a decision. In the first stage sample number is considered as $n_1 p_1$. Again we draw second sample of size n_2 and inspect the units. In this stage, there are n_1 *and* n_2 units are inspected and the corresponding probability of not taking any decision is $(n_1 + n_2)(1 - p_1)$.

By adding the above two sample sizes and get the required ASN.

$$\therefore ASN = n_1 p_1 + (n_1 + n_2)(1 - p_1)$$

$$= n_1 p_1 + n_1(1 - p_1) + n_2(1 - p_1)$$

$$ASN = n_1 + n_2(1 - p_1)$$

By taking the values of p along x axis and ASN along y axis, we get ASN curve.

ATI curve for Double Sampling Plan

Consider a double sampling plan with usual notations as $\begin{pmatrix} N \\ n_1 \\ c_1 \\ n_2 \\ c_2 \end{pmatrix}$. We select a sample of size n_1 and inspect the units. Let P_{a1} be the probability of acceptance of the lot on the basis of first sample. Here, the number of inspected items is taken as

$$n_1 P_{a1} \tag{1}$$

Suppose the lot is not accepted on the basis of first sample, we draw second sample of size n_2. Let P_{a2} be the probability of acceptance on basis of second sample when the first sample is already inspected. In the second stage, the number of inspected items for both samples is considered as

$$(n_1 + n_2)P_{a2} \tag{2}$$

Suppose the lot is rejected after the second sampling is inspected, we apply 100% inspection for the whole lot of the size N. In this stage, the probability of rejection is taken as $1-P_a$. In the rejection level, number of inspected item is

$$N(1-P_a) \tag{3}$$

By adding the above stated inspected items, we get required expression for ATI curve.

$$\therefore ATI = n_1 P_{a1} + (n_1 + n_2)P_{a2} + N(1-P_a)$$

$$= n_1 P_{a1} + (n_1 + n_2)(P_a - P_{a1}) + N(1-P_a)$$

$$= n_1 P_{a1} + (n_1 + n_2)[(1-P_{a1}) - (1-P_a)] + N(1-P_a)$$

$$ATI = n_1 + n_2(1-P_{a1}) + [N-(n_1+n_2)](1-P_a)$$

For different values of lot quality, we have to calculate probability of acceptance for first sample, combined sample and overall probabilities of acceptance P_a, by using lot quality p and the corresponding probabilities of acceptance P_a, we draw we required ATI curve which shows the mirror image of OC curve.

Problem 1:

A single sampling plan uses a sample size of 15 and an acceptance number 1. Using hypergeometric probabilities, compute the probability of acceptance of lots of 50 articles 2% defective.

Solution:

Given

N=50, n=15, c=1, p' =0.02 (2%)

Number of defective articles = 0.02*50=1

Number of non-defective article = 50-1=49

$$P_0 = \frac{49C_{15}}{50C_{15}}$$

$$= \frac{49!}{34! \times 15!} \times \frac{35! \times 15!}{50!} = 0.7$$

$$P_1 = \frac{1C_1 \times 49C_{14}}{50C_{15}}$$

$$= \frac{49!}{35! \times 14!} \times \frac{35! \times 15!}{50!} = 0.3$$

Probability of acceptance = 0.7+0.3=1.

Problem 2:

The lot size N is 2000 in a certain AOQL inspection procedure. The desired AOQL of 2% can be obtained with any one of the three sampling plans. These are:

(i) N=65, c=2
(ii) N=41, c=1
(iii) N=18, c=0

If a large number of lots 0.3%defective are submitted for acceptance, what will be the average number of units inspected per lot under each of these sampling plans?

Solution:

(i) Now

$$np' = \frac{65 \times 0.3}{100} = 0.195$$

From table G, for $np' = 0.195$ and c=2, $P_a = 0.999$

Average number of items inspected per lot $= P_a.n + (1 - P_a)N$

= 0.999*65+0.001*2000=66.935 (say 67)

(ii) $np' = \frac{41 \times 0.3}{100} = 0.123$

From table G, for $np' = 0.123$ and c=1, $P_a = 0.993$

Average number of items inspected per lot

=0.993*41+0.007*2000=54.959 (say 55)

(iii) $np' = \frac{18 \times 0.3}{100} = 0.054$ for $np' = 0.054$ and c=0, $P_a = 0.947$

Average number of items inspected per lot

=0.947*18+0.053*2000=123.046 (say 124)

Problem 3:

(a) Explain the step by step for constructing the OC curve for a single sampling plan.
(b) Draft the OC curve of the single sampling plan: n=300, c=5.

Solution:

(a) The step by step method of constructing the OC curve for a single sampling plan is as follows:

1. Step up table headings and the P_a column as follows:

n	np'	p'	P_a	$P_a p'$
			0.98 0.95 0.70 0.50 0.20 0.05 0.02	

where, n=sample size, np' =number of defectives, p' =fraction defective, P_a =probability of acceptance, $P_a p'$ =AOQ=Average Outgoing Quality.

The chosen of P_a will give ordinate values which, when co-ordinated with p' values to be derived, will facilitate construction of an OC curve.

2. Search Table G under the given np' value until the desired P_a (or close value to desired P_a) is located.

   (If the exact value is not found, the value in the P_a column should be changed to correspond with the one selected.

3. Place the np' value associated with the selected P_a in the np' column.
4. Divide the np' value by n. This will give the p' co-ordinate of P_a for the OC curve.

(b) To draft the OC curve for the single sampling plan, n=300, c=5.

1. **Table construction**

n	np'	p'	P_a	$P_a\,p'$
300			0.98	
300			0.95	
300			0.70	
300			0.50	
300			0.20	
300			0.05	
300			0.02	

2. **Finding np' and p'**

Search through Table, under np' =5 discloses P_a value of 0.983. This is the closest value to 0.98. The np' value associated with a P_a value of 0.983 is 2.0. This value of np' , when divided by n=300 gives p' value of 0.0067. The same procedure is followed for each of the other P_a values until the table is computed.

N	np'	p'	P_a	$P_a\,p'$
300	2.0	0.0067	0.983	0.0065
300	2.6	0.0087	0.951	0.00827
300	4.4	0.0147	0.72	0.0106
300	5.6	0.0187	0.512	0.00957
300	7.8	0.025	0.210	0.00526
300	10.5	0.035	0.05	0.00175
300	12.0	0.02	0.02	0.0008

The $P_a\,p'$ column is provided to give the necessary values for the graphical of an AOQ curve with p' being the abscissa and $P_a\,p'$ the ordinate.

Exercises

1. State the advantages and limitation of acceptance sampling over 100% inspection.
2. Explain the OC curve with reference to sampling inspection and the meaning of the term:
   (i) AQL
   (ii) LTPD
   (iii) Producer's risk
   (iv) Consumer's risk
3. State difference between Single Sampling plan and Double Sampling plan.
4. What is ATI? How will you compute the ATI for single sampling and double sampling plans?
5. Describe the principle of Single sampling plan.
6. Describe the principle of double sampling plan.
7. Explain the principles of double sampling plan and construct OC, ASN and ATI curves.

Unit - V

Variable Sampling Plan

5.1. Introduction

5.2. Types of Variable Sampling Plan

5.3. Variable Sampling Plan for a Process Parameter

5.4. Variable Sampling Plans to Estimate the Lot Percent Nonconforming

5.1. Introduction

Acceptance sampling by variable (or) variable sampling plan can be used if a quality characteristic is measured on a numeric scale or continuous scale and is known to follow some statistical distribution. This type of plan is based on sample measurements of the mean and standard deviation.

Advantages

1. For a given quality protection, smaller samples may be used in variable sampling plan than that with attribute sampling plan.
2. Variables information gives a better basis for quality improvements.
3. The conformance and non-conformance are used to estimate quality characteristic.
4. Variable information may give better basis for quality in acceptance decisions.
5. Errors of measurements are disclosed in variable information.
6. The primary advantage of using variable sampling plan is that more information can be obtained about the quality characteristic than when using an attribute sampling plan.

Disadvantages

1. The main disadvantage is that a separate sample plan is needed for each quality characteristic being investigated.
2. In variable sampling plan, for each quality characteristic we have to assume a distribution. In some cases, the assumed distribution and actual distribution are different in nature. This implies that quality protection may occur differently.
3. There is a possibility that a lot may be rejected even if it does not contain any defective items. This may cause some disagreements between customer and business man.

The Normality Assumption

When using acceptance sampling by variables procedures, it is generally assumed that the samples come from a normal population that is, the measured quality characteristic is assumed to be normally distributed. Consequently, a functional relationship exists between the measured quality characteristic and its mean and standard deviation.

5.2. Types of Variable Sampling Plans

There are two types of variable sampling plans. One type is used to control a process parameter and the other type is used to control the lot percent defectives. Variable sampling plans for a process parameter are designed to control the mean or standard deviation of a quality characteristic. Variable sampling plans for lot percent defectives are designed to determine the proportion of product that is in excess of a specified limit.

1. **Variable Sampling Plans for a process parameter**

(a) Variable Sampling Plans to estimate the process average with a single specification limit when σ known
(b) Variable Sampling Plans to estimate the process average with double specification limits when σ known
(c) Variable Sampling Plans to estimate the process average with a single specification limits when σ unknown
(d) Variable Sampling Plans to estimate the process average with double specification limits when σ unknown

2. **Variable Sampling Plans to estimate the lot percent defectives**

(a) Variable Sampling Plans with a single specification limit for large sample size when σ unknown
(b) Variable Sampling Plans with a single specification limit for large sample size when σ known
(c) Variable Sampling Plans with a double specification limit for large sample size when σ unknown
(d) Variable Sampling Plans with a double specification limit for large sample size when σ known

5.3. Variable Sampling Plan for a Process Parameter

Variable sampling plans for a process parameter are most likely to be used in sampling products that are submitted in bulk (for example, bags, and boxes). This type of sampling is concerned with the average quality of the product or with the variability in its quality.

(a) Variable Sampling Plans to Estimate the Process Average with a Single Specification Limit – σ Known

To design a single sampling plan for a process average with a single acceptance limit, four characteristics of the plan need to be identified. They are $\overline{X}_1$ (the average value of the quality characteristic for which the probability of acceptance is high), $\overline{X}_2$ (the average value of the quality characteristic for which the probability of acceptance is low), α (the probability of rejecting a lot that meets the specified quality level) and β (the probability of accepting a lot that does not meet the specified quality level). The OC curve of the single sampling plan should pass through two specified points $(1-\alpha, \overline{X}_1)$ *and* $(\beta, \overline{X}_2)$. The single sampling plan

can then be derived as follows. First, obtain the value of standard normal deviate so that the area under the standard normal curve is (1-α) call this value Z_1. Likewise, obtain the value of the standard normal deviate for β call this Z_2. Let $\overline{X}_a$ be the acceptance limit. By the normality assumption, we have

$$Z_1 = \frac{\overline{X}_1 - \overline{X}_a}{\sigma/\sqrt{n}} \tag{1}$$

$$Z_2 = \frac{\overline{X}_2 - \overline{X}_a}{\sigma/\sqrt{n}} \tag{2}$$

It is assumed, in this case, that the standard deviation σ is known and constant. Notice that the equations have two unknown variables $\overline{X}_a$ and n. Values for these variables can be obtained by solving the two equations (1) and (2) simultaneously. The resulting equations for n and $\overline{X}_a$ are

$$n = \left[\frac{(Z_2 - Z_1)\sigma}{\overline{X}_2 - \overline{X}_1}\right]^2 \tag{3}$$

and

$$\overline{X}_a = \frac{Z_2\,\overline{X}_1 - Z_1\,\overline{X}_2}{Z_2 - Z_1} \tag{4}$$

If the resulting value for n is not an integer, it should be rounded up to the next higher integer value.

Construction of an OC curve

The OC curve of a single sampling plan for a process with a single acceptance limit is constructed by plotting various values of the process average versus the probability of acceptance of each value. The probability of acceptance for a process average $\overline{X}$ under a sampling plan with parameters $\overline{X}_a$ and n is obtained by calculating Z, the standard normal deviate, as

$$Z = \frac{\overline{X} - \overline{X}_a}{\sigma/\sqrt{n}} \tag{5}$$

if an upper acceptance limit is required or

$$Z = \frac{\overline{X}_a - \overline{X}}{\sigma/\sqrt{n}} \tag{6}$$

if a lower acceptance limit is required. The area under the standard normal curve that is less than the value of normal deviate is the probability of acceptance value.

Problem 1:

Suppose that a company wants to set up a variables sampling plan for steel bars based on tensile strength. The company would like to accept steel bars with an average tensile strength of 10000 psi or higher 95% of the time. Conversely, the company would like to reject steel bars with an average tensile strength of 9950 psi or less 90% of time. In other words $\overline{X}_1 = 10000, \overline{X}_2 = 9950, \alpha = 0.05$ *and* $\beta = 0.10$. Suppose that the standard deviation is known to be 100 psi.

Solution:

The values of the standard normal deviates for $(1-\alpha)$ and β are 1.645 and -1.282 respectively. Substituting these values into given equations

$$n = \left[\frac{(Z_2 - Z_1)\sigma}{\overline{X}_2 - \overline{X}_1}\right]^2$$

$$n = \left[\frac{(-1.282 - 1.645)100}{9950 - 10000}\right]^2 = 34.27$$

and

$$\overline{X}_a = \frac{Z_2\,\overline{X}_1 - Z_1\,\overline{X}_2}{Z_2 - Z_1}$$

$$\overline{X}_a = \frac{-1.282(10000) - 1.645(9950)}{-1.282 - 1.645} = 9972$$

Rounding the value of n to the next higher integer gives n=35. The variables sampling plan is as follows. Take a random sample of 35 steel bars from each lot and compute the average tensile strength. If the average tensile strength of the sample is 9972 psi or higher accept the lot, otherwise reject it.

(b) Variable Sampling Plans to Estimate the Process Average with Double Specification Limits - σ is known

In statistical hypothesis, the sample statistics and population parameters are used to test the validity of the population parameter is not known we must use sample statistic. On the other hand if population parameters are known (given), we use the given parameters.

Variable sampling plan with specified upper limit and lower limit are specified is constructed by using some characteristics. In this case variable sampling plan consist of five characteristics and they are $\alpha, \beta, \overline{X}_1, \overline{X}_{2U}$ *and* $\overline{X}_{2L}$. Here α is the probability of rejecting a lot that means the specified quality level (producer's risk), β is the probability of accepting a lot that does not meet the specified quality level (consumer's risk). $\overline{X}_{2U}$ *and* $\overline{X}_{2L}$ are upper and lower values of $\overline{X}_2$ respectively. Let z_1 *and* z_2 be standard normal variates corresponding to

$1-\frac{\alpha}{2}$ *and* $1-\frac{\beta}{2}$ respectively. Our aim is to estimate $\bar{X}_U, \bar{X}_L$ and n. We know that the standard error of sample average is $\frac{\sigma}{\sqrt{n}}$.

$$\text{(i.e.) } SE(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

For estimating the unknown, we have to form four standard normal variables by using specification limits.

$$z_1^{\,U} = \frac{\bar{X}_U - \bar{X}_1}{\sigma/\sqrt{n}} \tag{1}$$

$$z_1^{\,L} = \frac{\bar{X}_1 - \bar{X}_L}{\sigma/\sqrt{n}} \tag{2}$$

$$z_2^{\,U} = \frac{\bar{X}_{2U} - \bar{X}_U}{\sigma/\sqrt{n}} \tag{3}$$

$$z_2^{\,L} = \frac{\bar{X}_L - \bar{X}_{2L}}{\sigma/\sqrt{n}} \tag{4}$$

There are four equations and three unknown variables $\bar{X}_U, \bar{X}_L$ and n. Now subtract equation (2) from equation (3),

$$(3)-(2) \Rightarrow$$

$$z_2^{\,U} - z_1^{\,L} = \frac{1}{\sigma/\sqrt{n}}\left[\bar{X}_{2U} - \bar{X}_U - \left\{\bar{X}_1 - \bar{X}_2\right\}\right]$$

$$\left[z_2^{\,U} - z_1^{\,L}\right)\frac{\sigma}{\sqrt{n}} = \bar{X}_{2U} - \bar{X}_U - \bar{X}_1 + \bar{X}_L \tag{5}$$

$$Again \frac{(3)}{(2)} \Rightarrow$$

$$\frac{z_2^{\,U}}{z_2^{\,L}} = \frac{\left(\bar{X}_{2U} - \bar{X}_U\right)/\left(\sigma/\sqrt{n}\right)}{\left(\bar{X}_1 - \bar{X}_L\right)/\left(\sigma/\sqrt{n}\right)}$$

$$\frac{z_2^{\,U}}{z_2^{\,L}} = \frac{\bar{X}_{2U} - \bar{X}_U}{\bar{X}_1 - \bar{X}_L} \tag{6}$$

$$\frac{(4)}{(2)} \Rightarrow \frac{z_2^{\,L}}{z_1^{\,L}} = \frac{\left(\bar{X}_L - \bar{X}_{2L}\right)/\left(\sigma/\sqrt{n}\right)}{\left(\bar{X}_1 - \bar{X}_L\right)/\left(\sigma/\sqrt{n}\right)}$$

$$= \frac{\bar{X}_L - \bar{X}_{2L}}{\bar{X}_1 - \bar{X}_L} \tag{7}$$

The left hand sides of equation (6) and (7) are considered as identical and implies that the RHS are also identical.

$$\frac{\overline{X}_{2U}-\overline{X}_{U}}{\overline{X}_{1}-\overline{X}_{L}}=\frac{\overline{X}_{L}-\overline{X}_{2L}}{\overline{X}_{1}-\overline{X}_{L}}$$

$$\Rightarrow \overline{X}_{2U}-\overline{X}_{U}=\overline{X}_{L}-\overline{X}_{2L}$$

$$\overline{X}_{2U}+\overline{X}_{2L}=\overline{X}_{L}+\overline{X}_{U}$$

$$2\overline{X}_{1}=\overline{X}_{U}+\overline{X}_{L} \qquad (8)$$

$$(2)-(3)\Rightarrow$$

$$z_1{}^{L}-z_2{}^{U}=\frac{1}{(\sigma/\sqrt{n})}\left[\overline{X}_{1}-\overline{X}_{L}-(\overline{X}_{2U}-\overline{X}_{U})\right]$$

$$\left(z_1{}^{L}-z_2{}^{U}\right)\sigma/\sqrt{n}=\overline{X}_{1}-\overline{X}_{L}-\overline{X}_{2U}+\overline{X}_{U}$$

$$\left(X_1{}^{L}-z_2{}^{U}\right)\sigma/\sqrt{n}+\overline{X}_{2U}-\overline{X}_{1}=\overline{X}_{U}-\overline{X}_{L} \qquad (9)$$

By solving equations (8) and (9) and get the value of $\overline{X}_{U}$

$$(8)\ \Rightarrow \overline{X}_{U}+\overline{X}_{L}=2\overline{X}_{1}$$

$$(9)\ \Rightarrow \overline{X}_{U}-\overline{X}_{L}=(z_1{}^{L}-z_2{}^{U})\frac{\sigma}{\sqrt{n}}+\overline{X}_{2U}-\overline{X}$$

Adding these two equations and get,

$$2\overline{X}_{U}=[z_1{}^{L}-z_2{}^{U})\frac{\sigma}{\sqrt{n}}+\overline{X}_{2U}+\overline{X}_{1}$$

$$\overline{X}_{U}=\frac{1}{2}\left[\left(z_1{}^{L}-z_2{}^{U}\right)\frac{\sigma}{\sqrt{n}}+\overline{X}_{2U}+\overline{X}_{1}\right]$$

Substitute the value of $\overline{X}_{U}$ in equation (8) and get

$$\overline{X}_{U}+\overline{X}_{L}=2\overline{X}_{1}$$

$$\Rightarrow \overline{X}_{L}=2\overline{X}_{1}-\overline{X}_{U}$$

$$=2\overline{X}_{1}-\frac{1}{2}\left[\left(z_1{}^{L}-z_2{}^{U}\right)\frac{\sigma}{\sqrt{n}}+\overline{X}_{2U}+\overline{X}_{1}\right]$$

$$=\frac{3}{2}\overline{X}_{1}-\frac{1}{2}\left[\left(z_1{}^{L}-z_2{}^{U}\right)\frac{\sigma}{\sqrt{n}}+\overline{X}_{2U}\right]$$

$$\bar{X}_L = \frac{1}{2}\left[3\bar{X}_1 - \left\{\left(z_1{}^L - z_2{}^U\right)\frac{\sigma}{\sqrt{n}} + \bar{X}_{2U}\right\}\right]$$

The equations (5), (10) and (11) give the required values of n, $\bar{X}_U$ *and* $\bar{X}_L$ respectively.

Problem 2:

Suppose that a company would like to design a variable sampling plan for steel bars based on tensile strength with both an upper and lower acceptance limit. The company does not wish to accept steel bars with a tensile strength greater than 10100 psi or lower than 9900 psi more than 10% of the time. In other words $\bar{X}_{2L} = 9900, \bar{X}_{2U} = 10000$ *and* $\beta = 0.10$. Suppose that $\bar{X}_1 = 10000, \alpha = 0.05$ and the standard deviation is known to be 100 psi.

Solution:

By using the equations, the following equations are obtained

$$z_1{}^U = \frac{\bar{X}_U - \bar{X}_1}{\sigma/\sqrt{n}}$$

$$1.960 = \frac{\bar{X}_U - 10000}{100/\sqrt{n}}$$

$$z_1{}^L = \frac{\bar{X}_1 - \bar{X}_L}{\sigma/\sqrt{n}}$$

$$1.960 = \frac{10000 - \bar{X}_L}{100/\sqrt{n}}$$

$$z_2{}^L = \frac{\bar{X}_L - \bar{X}_{2L}}{\sigma/\sqrt{n}}$$

$$1.645 = \frac{\bar{X}_L - 9900}{100/\sqrt{n}}$$

$$z_2{}^U = \frac{\bar{X}_{2U} - \bar{X}_U}{\sigma/\sqrt{n}}$$

$$1.645 = \frac{10100 - \bar{X}_U}{100/\sqrt{n}}$$

Since the last equation can be derived from the other three equations, it will not be considered in the computations. Using the remaining three equations and solving for $\bar{X}_U, \bar{X}_L$ and n simultaneously, we obtain $\bar{X}_U = 10054, \bar{X}_L = 9946$ and n=13. In summary, the variables sampling plan is as follows. Obtain a random sample of 13 steel bars from a lot and compute

the average tensile strength. If the sample average is higher than 10054 psi or lower than 9946 psi reject the lot, otherwise accept it.

(c) Variable Sampling Plans to Estimate the Process Average with a Single Specification Limits - σ unknown

Earlier, it was assumed that the variance is known. Consider the case where the variance is unknown. In this situation, a larger sample size will generally be required for a sampling plan than for a corresponding sampling plan in which the variance is known. The reason is that an additional amount of uncertainty is introduced into the sampling procedure, hence the need for a larger sample size. It is assumed that the items from a process or lot are still normally distributed.

Since the variance is unknown, the deviation of the sampling plan will be based on the t statistic. The t statistic was given equation

$$t_0 = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

in slightly different form as

$$t = \frac{\overline{X} - \overline{X}_1}{S/\sqrt{n}}$$

$$S^2 = \frac{\sum_{i=1}^{n} X_i^{\,2} - n\overline{X}^{\,2}}{n-1}$$

and X_i is the measured quality characteristic of the i^{th} unit in the sample. Notice that the t statistic is expressed in terms of S, which in turn is expressed in terms of n, an unknown parameter. Therefore, the parameters of the sampling plan cannot be obtained in terms of sample standard S. Notice also that the sample standard deviation is based on the sample obtained from a lot. Therefore, the OC curve for a sampling plan will vary from lot to lot. Thus, for the reasons mentioned previously, a sampling plan cannot be derived unless an estimate of the standard deviation can be made.

Consider the design of sampling plan with characteristics $\overline{X}_1, \overline{X}_2, \alpha \text{ and } \beta$. Let $\hat{\sigma}$ be the estimate of the standard deviation of the process or lot. Duncan (1986) provides several OC curves for single limit sampling plans based on the t statistic for α=0.05. To obtain the required sample size that meets the characteristics of our sampling plan, it is first necessary to compute λ as,

$$\lambda = \frac{\left|\overline{X}_1 - \overline{X}_2\right|}{\hat{\sigma}}$$

We can find n for $P_a = \beta$. Thus, the sampling plan is as follows:

Take a random sample of size n and compute the variate t to be

$$t = \frac{\overline{X} - \overline{X}_1}{S/\sqrt{n}}$$

If $t \le t_{0.05, n-1}$, reject the lot, otherwise accept it. Notice that even through an estimate of the standard deviation $\hat{\sigma}$ is used to derive a sampling plan, this estimate is not used during sampling. Instead, the standard deviation of a sample S is used to calculate for the variable t.

Problem 3:

Consider problem 1 with $\overline{X}_1 = 10000, \overline{X}_2 = 9950, \alpha = 0.05$ *and* $\beta = 0.10$. Assume that the standard deviation is unknown, but the company decides that a good estimate would be 90 psi (i.e. $\hat{\sigma} = 90$).

Solution:

By using the equation λ is computed to be

$$\lambda = \frac{\left|\overline{X}_1 - \overline{X}_2\right|}{\hat{\sigma}}$$

$$\lambda = \frac{\left|10000 - 9950\right|}{90} = 0.56$$

The t statistic for α=0.05 and 29 degrees of freedom is -1.699. Therefore, the sampling plan is as follows. Take a random sample of 30 steel bars from a lot and compute the sample average $\overline{X}$ and the sample standard deviation S. compute the variable t as

$$t = \frac{\overline{X} - 10000}{S/\sqrt{30}}$$

If $t \le -1.699$, reject the lot, otherwise accept it.

5.4. Variables Sampling Plans to Estimate the Lot Percent Nonconforming

It is assumed that the quality characteristic of interest is measurable on a continuous scale and that it is normally distributed, with a known and constant variance. By the normality assumption, a functional relationship exists between the lot percent nonconforming and the sample mean $\overline{X}$ and the standard deviation σ. This functional relationship is given by the standard normal deviates

$$Z_L = \frac{\overline{X} - L}{\sigma} \qquad (1)$$

If a lower limit L is specified or

$$Z_U = \frac{U - \overline{X}}{\sigma} \qquad (2)$$

If an upper limit U is specified. Therefore, the estimate of the lot percent nonconforming is the area under the normal curve that exceeds the normal deviate $Z_L or Z_U$. Notice that the normal deviate varies directly with the sample mean such that as the sample mean moves further away from the specification limit, the percent nonconforming decreases and vice versa.

Form 1 and Form 2

There are two methods by which a variables sampling plan that estimates the lot percent nonconforming can operate. They are called Form 1 (or the k-method) and Form 2 (or the M-method). Under Form 1, the standard normal deviate is computed by using equation (1) or (2) depending on whether a lower or an upper limit is specified. This method is called the k-method since the standard normal deviate $Z_L or Z_U$ is compared to critical value k. If $Z_L or Z_U$ is greater than or equal to the value of k, the lot is accepted. The operation of the variables sampling plan using this method is as follows. Take a random sample of size n from a lot. Compute the sample average $\overline{X}$ and the standard normal deviate $Z_L or Z_U$. If $Z_L or Z_U$ is greater than or equal to k, accept the lot, otherwise reject it.

The M-method is similar to k-method, but the estimation of the lot percent nonconforming is carried one step further. Let $\overline{X}$ be the sample mean the variance σ^2 is assumed to be known and constant. The normal deviate is computed as

$$Q_L = \frac{\overline{X} - L}{\sigma}\sqrt{\frac{n}{n-1}} \tag{1}$$

If a lower limit is specified or

$$Q_U = \frac{U - \overline{X}}{\sigma}\sqrt{\frac{n}{n-1}} \tag{2}$$

If an upper limit is specified. Notice that

$$Q_L = Z_L\sqrt{\frac{n}{n-1}} \tag{3}$$

and

$$Q_U = Z_U\sqrt{\frac{n}{n-1}} \tag{4}$$

The normal deviates $Q_L and Q_U$ provides a slightly better estimate of the lot percent nonconforming, since they are unbiased and have minimum variance. After $Q_L or Q_U$ is computed, the lot percent nonconforming p is estimated by determining the area under the normal curve that exceeds the quality index $Q_L or Q_U$. Call this estimate $\hat{p}$. If $\hat{p}$ exceeds a maximum allowable percent nonconforming M the lot is rejected, otherwise accepted. This method is called the M-method since the estimated lot percent nonconforming $\hat{p}$ is compared is as follows. Take a random sample of size n from a lot. Compute $\overline{X}$ and quality index

Q_L *or* Q_U. From the quality index, obtain the estimated lot percent nonconforming $\hat{p}$. If $\hat{p}$ is greater than M reject the lot, otherwise accept it.

(a) Variable Sampling Plans with a single specification limit for large sample size when σ unknown

In statistical hypothesis, when the population parameter is unknown, we have to use sample observations. Consider a sample of size n (>30) with the observations $x_1, x_2, ..., x_n$. By using the given observations, we have to calculate sample mean and sample standard deviation. Based on the computed sample statistics and specification limits, we have to frame variable sampling plan and estimate sample size n and critical value k. Let α, β, p_1 *and* p_2 be the four characteristics which are used to frame the variable sampling plan. Let z_α, z_β, z_1 *and* z_2 be the standard normal variables corresponding to the characteristics α, β, p_1 *and* p_2. In 1947, Eisenhart, Hastay and Wallis have derived the sample size and critical value as

$$n = \left(1 + \frac{k^2}{2}\right)\left(\frac{z_\alpha + z_\beta}{z_1 - z_2}\right)^2 \tag{1}$$

and

$$k = \frac{z_\alpha z_2 + z_\beta z_1}{z_\alpha + z_\beta} \tag{2}$$

For framing standard variables we have to compute sample mean $\bar{X}$ and sample standard deviation S. In addition that we have to fix lower limit (L) and upper limit (U). If an upper limit is specified

$$z_U = \frac{U - \bar{X}}{S} \tag{3}$$

If a lower limit is specified

$$Z_L = \frac{\bar{X} - L}{S} \tag{4}$$

If either z_U *or* z_L is greater than or equal to critical value k, then accept the lot. If it is not so, we reject the lot.

Construction of an OC Curve

As in the σ known case, the OC curve is constructed by plotting a range of lot percent nonconforming values p versus the probability of acceptance of each. Suppose that a single lower specification limit L is given. Let p be the percent nonconforming and let Z_p be the standard normal deviate in which the area is (1-p). The probability of acceptance for a p value is defined by calculating the normal deviate Z_a using the following equation:

$$Z_a = \frac{k - Z_p}{\sqrt{\frac{1}{n} + \frac{k^2}{2n}}} \tag{1}$$

The area exceeding Z_a can be determined. This area is the probability of acceptance value for p.

Problem 4:

Suppose a company is interested in setting up a variables sampling plan for thin-wall cylinders based on the tensile strength with $p_1 = 0.01, p_2 = 0.10, \alpha = 0.05 \, and \, \beta = 0.10$. Suppose also that the variance is unknown.

Solution:

The values of the standard normal deviates for $p_1, p_2, \alpha \, and \, \beta$ are $Z_1 = 2.327$, $Z_2 = 1.282, Z_\alpha = 1.645 \, and \, Z_\beta = 1.282$ respectively. Using the given equation to solve for k, we have

$$k = \frac{Z_\alpha Z_2 + Z_\beta Z_1}{Z_\alpha + Z_\beta}$$

$$k = \frac{(1.645)(1.282) + (1.282)(2.327)}{1.645 + 1.282} = 1.74$$

Using the given equation to solve for n, we have

$$n = \left(1 + \frac{k^2}{2}\right)\left(\frac{Z_\alpha + Z_\beta}{Z_1 - Z_2}\right)^2$$

$$n = \left(1 + \frac{(1.74)^2}{2}\right)\left(\frac{1.645 + 1.282}{2.327 - 1.282}\right)^2 = 19.72$$

Since n is integer valued, we round 19.72 to the next higher integer, which is 20. Therefore, the variables sampling plan is as follows. Take a random sample of 20 thin-wall cylinders and compute the average tensile strength and the sample standard deviation S. Using the given equation, to compute Z_L.

$$Z_L = \frac{\overline{X} - L}{S}$$

If $Z_L \geq 1.74$, accept the lot, otherwise reject it.

(b) Variable Sampling Plans with a single specification limit for large sample size when σ known

To derive a sampling plan to control the lot percent nonconforming, it is necessary to known four characteristics of the plan. They are α (the probability of rejecting a lot that does not meet the specified quality level), β (the probability of accepting a lot that does not meet the specified quality level), p_1 (the percent nonconforming value for which the probability of acceptance is high and p_2 (the percent nonconforming value for which the probability of acceptance is low). The OC curve of the sampling plan should pass through the two points $(1-\alpha, p_1)$ *and* (β, p_2). Let Z_α be the normal deviate for which the area under the standard normal curve is (1-α) and let Z_β be the normal deviate for which the area is (1-β). Let $\bar{X}$ be the sample average and suppose that a single lower limit L is specified and the standard deviation σ is known. Under Form 1, a lot will be accepted if the following holds

$$\frac{\bar{X}-L}{\sigma} \geq k$$

Adding and subtracting the term $\frac{\mu}{\sigma}$, we get

$$\frac{\bar{X}-L}{\sigma} + \frac{\mu}{\sigma} - \frac{\mu}{\sigma} \geq k$$

Such that

$$\frac{\bar{X}-\mu}{\sigma} + \frac{\mu-L}{\sigma} \geq k$$

Which can be written

$$\frac{\bar{X}-\mu}{\sigma} \geq k - \frac{\mu-L}{\sigma}$$

Multiplying both sides by $\sqrt{n}$, we get

$$\left(\frac{\bar{X}-\mu}{\sigma}\right)\sqrt{n} \geq \left(k - \frac{\mu-L}{\sigma}\right)\sqrt{n}$$

Which can be written

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \geq \left(k - \frac{\mu-L}{\sigma}\right)\sqrt{n}$$

Let Z_1 *and* Z_2 be the standard normal deviates such that the areas exceeding these values are p_1 *and* p_2 respectively. Therefore,

$$\frac{\mu_1 - L}{\sigma} = Z_1$$

and

$$\frac{\mu_2 - L}{\sigma} = Z_2$$

So that

$$P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq (k - Z_1)\sqrt{n}\right) = \alpha$$

and

$$P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq (k - Z_2)\sqrt{n}\right) = \beta$$

Since $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ is normally distributed, with mean 0 and variance σ^2, we have

$$(k - Z_1)\sqrt{n} = Z_\alpha \tag{1}$$

and

$$(k - Z_2)\sqrt{n} = Z_\beta \tag{2}$$

To specify a sampling plan, it is necessary to know the sample size n and the critical value k. These two parameters can be obtained by solving equations (1) and (2) simultaneously. The resulting equations are,

$$n = \left(\frac{Z_\beta + Z_\alpha}{Z_1 - Z_2}\right)^2 \tag{3}$$

and

$$k = Z_1 - \frac{Z_\alpha}{\sqrt{n}} \tag{4}$$

If α is to be used as specified or

$$k = Z_2 + \frac{Z_\beta}{\sqrt{n}} \tag{5}$$

If β is to be used as specified.

Under Form 2, it is necessary to compute $k\sqrt{n/(n-1)}$, where k is the critical value obtained previously. The critical value M is determined as the area under the normal curve that is in excess of the value $k\sqrt{n/(n-1)}$.

Construction of an OC Curve

The OC curve for a sampling plan that estimates the lot percent nonconforming can be constructed by plotting a range of lot percent nonconforming values p versus the probability of acceptance of each value.

Suppose that a lower limit is specified. Let p be the lot percent nonconforming and let Z_p be the corresponding standard normal deviate in which the area is (1-p). Let $\overline{X}_p$ be the sample average such that the following holds

$$\frac{\overline{X}_p - L}{\sigma} = Z_p$$

Thus the probability of acceptance is given by the following expression

$$P\left(\frac{\overline{X}_p - \mu}{\sigma/\sqrt{n}} \geq \left(k - Z_p\right)\sqrt{n}\right) = a$$

So that

$$Z_a = \left(k - Z_p\right)\sqrt{n}$$

The area in excess of Z_a is the probability of acceptance for p.

Problem 5:

Suppose that it is desired to set up a sampling plan for thin-wall cylinders based on the tensile strength. The lower specification limit is 170 psi. The sampling plan should have the characteristic such that a lot will be accepted with a probability of 0.95 if it contains 1% defective and will be rejected with probability of 0.90 if it contains 10% defective. In other words $p_1 = 0.01, p_2 = 0.10, \alpha = 0.05$ *and* $\beta = 0.10$. Suppose that the standard deviation is known to be 10 psi.

Solution:

The values of the standard normal deviates for p_1, p_2, α *and* β are $Z_1 = 2.327$, $Z_2 = 1.282, Z_\alpha = 1.645$ *and* $Z_\beta = 1.282$ respectively. Using the given equation to solve for n, we obtain

$$n = \left(\frac{Z_\beta + Z_\alpha}{Z_1 - Z_2}\right)^2$$

$$n = \left(\frac{1.282 + 1.645}{2.327 - 1.282}\right)^2 = 7.845$$

Since n is integer valued, we round 7.845 to the next higher integer, which is 8. Using the given equation, keeping α at the 0.05 level and solving for k, we obtain

$$k = Z_1 - \frac{Z_\alpha}{\sqrt{n}}$$

$$k = 2.327 - \frac{1.645}{\sqrt{7.845}} = 1.74$$

Therefore, the variables sampling plan under Form 1 will be as follows. Take a random sample of eight thin-wall cylinders from the lot and compute the sample average tensile strength. Using the given equation, compute Z_L. If $Z_L \geq 1.74$, accept the lot, otherwise reject it.

Under Form 2, an extra computational step is needed to obtain an estimate of the percent nonconforming. The value of M is obtained by the first computing

$$k\sqrt{\frac{n}{(n-1)}}$$

$$k = 1.74\sqrt{\frac{8}{7}} = 1.86$$

The area under the normal curve of this value is the value of M, which is 0.0314. Therefore, the sampling plan under Form 2 is as follows. Take a random sample of eight cylinders from the lot. Using the given equation, compute Q_L.

$$Q_L = \frac{\bar{X} - L}{\sigma}\sqrt{\frac{n}{n-1}}$$

Determine $\hat{p}_L$, the area in excess of Q_L. If $\hat{p}_L \leq 0.0314$, accept the lot, otherwise reject it.

Exercises

1. What are sampling plans by variables? Give the assumptions under which such plans are defined.
2. Derive the sampling plan to estimate process parameter with double specification limits when σ is known.
3. Derive the sampling plan to estimate process average with a single specification limit when σ is known.
4. What is meant by acceptance sampling by variables? Also, state its importance in industries.
5. Derive the variable sampling plans to estimate the process average with a single specification limits when σ is unknown.

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